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Stringy Models of Modified Gravity: Space-time defects and Structure Formation

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Starting from microscopic models of space-time foam, based on brane universes propagating in bulk space-times populated by D0-brane defects ("D-particles"), we arrive at effective actions used by a low-energy observer on the brane world to describe his/her observations of the Universe. These actions include, apart from the metric tensor field, also scalar (dilaton) and vector fields, the latter describing the interactions of low-energy matter on the brane world with the recoiling point-like space-time defect (D-particle). The vector field is proportional to the recoil velocity of the D-particle and as such it satisfies a certain constraint. The vector breaks locally Lorentz invariance, which however is assumed to be conserved on average in a space-time foam situation, involving the interaction of matter with populations of D-particle defects. In this paper we clarify the role of fluctuations of the vector field on structure formation and galactic growth. In particular we demonstrate that, already at the end of the radiation era, the (constrained) vector field associated with the recoil of the defects provides the seeds for a growing mode in the evolution of the Universe. Such a growing mode survives during the matter dominated era, provided the variance of the D-particle recoil velocities on the brane is larger than a critical value. We note that in this model, as a result of specific properties of D-brane dynamics in the bulk, there is no issue of overclosing the brane Universe for large defect densities. Thus, in these models, the presence of defects may be associated with large-structure formation. Although our string inspired models do have (conventional, from a particle physics point of view) dark matter components, nevertheless it is interesting that the role of "extra" dark matter is also provided by the population of massive defects. This is consistent with the weakly interacting character of the D-particle defects, which predominantly interact only gravitationally.

I. INTRODUCTION AND MOTIVATION

Relativistic modified gravity theories (*e.g.* models in Refs. [1, 2]) have been presented originally as field theoretic alternatives to dark matter models, offering support to the heuristic Modified Newtonian Dynamics approach [3]. Although, at least in their simplest form, such models may be ruled out by means of recent precision astrophysical measurements using gravitational lensing [4], nevertheless some of their features may characterise microscopic theories of quantum gravity, in harmonic co-existence with substantial dark matter components, thereby avoiding the above-mentioned stringent constraints.

For instance, it has been shown in Ref. [5], that the bi-metric nature of the TeVeS (Tensor-Vector-Scalar) model, as well as the existence of a Lorentz-violating vector field A_μ, do characterise the low-energy limit of certain string-theory models of space-time foam [6, 7], involving three-brane ("D3-brane", a three-space-dimensional Dirichlet brane) worlds [8, 9] embedded in higher-dimensional (bulk) space-times punctured by D0-brane ("D-particle") defects. In such models, the D3-brane representing our Universe, which may be obtained from appropriate compactification of a higher-dimensional brane, moves in the bulk and in this way the D-particles cross it, resembling — from a D3-brane observer viewpoint — flashing "on" and "off" foamy structures ("D-foam"). There are topologically non-trivial interactions of open string states, attached on the D3-brane and representing ordinary matter, with such defects, involving splitting of the initial string and creation of intermediate string states, stretched between the D-particle and the D3-brane universe, exhibiting length oscillations. Such processes result in local distortions of the neighbouring space-time [6]

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \ , \quad h_{0i} = v_i = g_{\rm s}\frac{\Delta k_i}{M_{\rm s}} \ , \tag{1}$$

to leading order in the recoil velocities $v_i \ll 1$, due to recoil of the D-particle defect to conserve energy and momentum during the scattering. In the above formula, $g_{\rm s}$ is the (weak) string coupling, $M_{\rm s}$ is the string mass scale, which is in

general different from the four-dimensional Planck scale $M_{\rm P} = 1.2. \times 10^{19}$ GeV, and v_i is the recoil three-velocity of the D-particle, with Δk_i the momentum transfer of the open string state.

Due to electric charge conservation, only electrically neutral matter interactions with the D-particles [1] are allowed. In this sense, there is a naturally induced *bi-metric* structure in the model, given that it is only the electrically neutral excitations of the low-energy effective theory that predominantly feel the induced metric Eq. (1). Moreover, like in TeVeS models, there exist *vector field* structures, provided by the recoil velocities of the D-particles, which break Lorentz invariance locally.

As pointed out by the authors of Ref. [11], it is the vector field of the TeVeS theory that they were working with, which plays the crucial role in reproducing the observed power spectrum by generating a cosmological instability that produces the large cosmic structures of today's Universe. It is important to note that the arguments on the role of the vector field in reproducing large scale structures and the correct phenomenology seem to be generic, in the sense that they do not depend on the detailed Lagrangian of the original TeVeS theory proposed by Bekenstein [1]. More precisely, it is only the basic important features of the theory, namely the existence of the two metrics (bi-metric theory) and of the (Lorentz-violating) "aether-like" vector field, that appear to be important in this respect.

The basic aim of this paper is to extend the link between the above mentioned features and the deduced cosmological perturbation theory, in the framework of some specific backgrounds of string theory proposed some time ago [6] as consistent candidates for a quantum space-time foam background in string theory. Specifically, we shall construct a low-energy effective action, describing (partly though) the low-energy aspects of such a stringy space-time foam, and argue that it acquires the form of a modified gravity theory, with scalar and vector fields arising naturally, as a result of the interaction of matter with the D-particle space-time defects in the foam. Then, we shall consider the equations for cosmological perturbations, solve them to linear order for small perturbations, and argue that the vector fields in the theory play a non-negligible role in structure formation, in the spirit of Ref. [11]. However, we stress that such theories do not provide alternative to dark matter, given that the latter exists naturally in the superstring-inspired models we are working with, represented by stable superpartners to the standard model matter that characterises the low-energy excitations of the models. In this sense, our approach is entirely different from models which are alternatives to dark matter [1, 2]. Nevertheless, the D-particle defects themselves play a role analogous to dark matter, given that their recoil furnishes the Universe with an effective vector field, whose perturbations in late eras of the Universe (radiation and matter dominated) do posses growing modes and can participate non-trivially in large structure formation. Thus the massive D-particles can, in addition to baryonic and other type of matter, contribute to galaxy and galactic cluster formation.

To set-up the framework we shall use, we briefly review in Section II the brane D-foam model that will serve as our microscopic framework for discussing modified gravity in the low energy limit. First we present the underlying formalism of the world-sheet deformation that describes the interaction of an open string state with a recoiling D-particle and derive the associated metric distortion of the neighbouring space-time, due to the recoil of the defect. We explain in detail the emergence of a vector "gauge" field A_μ as a result of this interaction, which will play a crucial role in our analysis in providing us with the growing mode that can lead to structure formation. The vector fields are proportional to the recoil velocities of the D-particles, and as such they satisfy a certain type of constraint (*cf.* Eq. (52)), which is crucial for the appearance of the growing mode. We discuss in Section III more general background configurations for the dilaton, graviton and gauge fields, in a way consistent with the conformal invariance of the world-sheet of the string. We first present the Dirac-Born-Infeld (DBI) action on 3-brane worlds and we then proceed with space-time curvature correction terms in the DBI action. In Section IV we study the four-dimensional induced effective action on a D3-brane world for the gravitational, dilaton and gauge fields. This is the action that we subsequently use to analyse the role of vector perturbations in structure formation. In Section V we derive the equations of motion for the vector, metric and dilaton fields and consider their cosmological perturbations. In Section VI we discuss the appearance of a growing mode in both the radiation and matter dominated eras of the brane Universe, provided a sufficiently large variance of recoil velocities of the D-particles exists. In this sense, the defects act as a dark matter component, which is in addition to the conventional (from a particle physics viewpoint) dark matter components of string effective models, e.g. supersymmetric partners of matter or graviton fields. Finally in Section VII we give our conclusions and outlook.

We use the following conventions: for the metric we use the signature $(-,+,+,+)$ and the Riemann curvature tensor is defined as $R^\alpha_{\beta\,\gamma\,\delta} = \partial_\delta\,\Gamma^\alpha_{\ \beta\,\gamma} + \Gamma^\lambda_{\ \beta\,\gamma}\,\Gamma^\alpha_{\ \lambda\,\delta} - (\gamma \longleftrightarrow \delta)$.

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[1] This is the case of type IIA string theory, which allows point-like branes. In the phenomenologically more realistic type IIB string models, there are no D0-brane configurations allowed. In such a case, one can still construct consistent D-foam models [10] in which the role of the D-particle is played by D3-branes wrapped up around small three cycles. In such a case, electrically charged excitations do interact non-trivially with the D-foam, but these interactions are significantly suppressed compared with those of the neutral excitations.

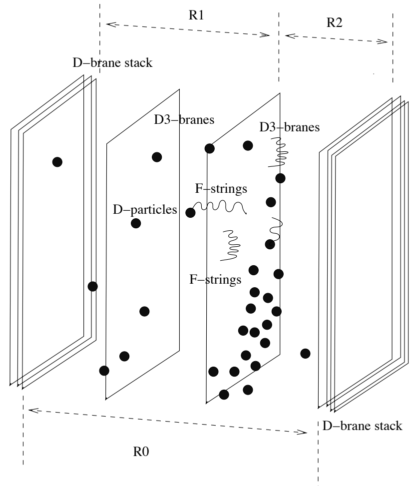

FIG. 1: Schematic representation of a generic D-particle space-time foam model. The model of Ref. [6], which acts as a prototype of a D-foam, involves two stacks of D8-branes, each stack being attached to an orientifold plane.

II. REVIEW OF D-FOAM INSPIRED MODIFIED GRAVITY EFFECTIVE THEORIES

The prototype model that we shall use as a microscopic motivation of our considerations in this work is the so-called D-brane model of space-time "foam", in which our Universe is represented as a (compactified) three-brane world moving in a higher-dimensional bulk space-time which is populated by point-like D0-brane defects (D-particles). Such a model is appropriate for type IIA string theory and the associated supergravity effective field theory in the bulk and was first presented in Ref. [6]. There is a relative motion of the defects with respect to the branes, which implies that, as the brane universe moves in the bulk, the D-particles cross it, so that from the point of view of an observer on the brane they look like "flashing on and off" foamy structures (hence the terminology "D-particle foam" model). In string perturbation theory, it can be shown that for an adiabatic relative motion between a D-particle defect and the brane, with the defect moving perpendicularly to the brane, the induced potential on the brane, due to open strings stretched between the D-particle and the brane, depends on both the relative velocity $v_\perp$ and the distance of the bulk defect from the brane:

$$\mathcal{V}^{\rm long}_{D0-D8} = + \frac{r\,(v_\perp^{\rm long})^2}{8\pi\alpha'}\ ,\ r \gg \sqrt{\alpha'} \tag{2}$$

However, a D-particle close to the D3-brane (compactified D8), at a distance $r' \ll \sqrt{\alpha'}$, moving adiabatically in the perpendicular direction with a velocity $v_\perp^{\rm short}$,will induce the potential :

$$\mathcal{V}^{\rm short}_{D0-D8} = - \frac{\pi\alpha' (v_\perp^{\rm short})^2}{12 r'^3}. \tag{3}$$

This difference in sign, implies that, one can arrange for the densities of far away and nearby bulk D-particles, which are not in general homogeneous, to be such that the total contribution to the brane world's vacuum energy is always sub-critical, so that issues such as *over-closure* of the Universe by a significant population of D-particle defects can be *avoided* [7]. Consequently, the astrophysical data can only constrain the total energy density on the brane due to the defects, that is the algebraic sum of the above two contributions in Eqs. (2), (3). Hence, the density of defects on the brane remains unconstrained. We shall come back to this point in Section VI.

These considerations imply that, at least at early eras of the Universe, we can plausibly assume that the brane Universe passes through bulk regions which are densely populated by D-particles, in such a way that there is a sufficiently high density of D-particles on the brane, such that the particle excitations, represented by open strings with their ends attached on the brane, propagate in a "*medium*" of such D-particle defects. Topologically non-trivial interactions between the defects and matter strings can occur, involving the capture and splitting of open strings by the defects and creation of stretched strings between the defects and the brane during such processes [6, 7] (*cf.*

Fig. 1). The capture process is represented from a world-sheet point of view as an impulse which leads to a metric deformation of the neighbouring space-time due to the recoil of the D-particle. The induced metric distortions depend on both the space-time coordinates and the momentum transfer of the matter particle during its scattering with the defect. The recoil of the defect breaks locally Lorentz invariance of the space-time as a result of its momentum direction, and in the effective low-energy limit is described as a vector field A_μ, which should not be confused with the electromagnetic field. On average, over large populations of defects Lorentz invariance may be restored in the sense of having a zero vacuum expectation value of the vector field, but non-zero variances. The purpose of this work is to examine the effective low-energy target space action describing the interaction of matter excitations with the D-particles, through the field A_μ and discuss in this context the potential role of the perturbations of the vector field in inducing a growing mode that could participate in large-scale structure formation at redshifts of order $z \sim 1$. As a result of charge conservation, charged open string excitations cannot interact with the (neutral) defects via the above-mentioned splitting. Only electrically neutral excitations are therefore interacting predominantly with the D-particle foam. Cosmologically it is therefore the neutrinos and photons that feel mostly the effects of the foam at these early eras, and hence it is these particles that can find themselves propagating in the background of recoiling D0-branes, and hence of the associated vector field A_μ. We shall show here that D-particles can play the role of a dark matter component, in addition to either supersymmetric partners of standard model excitations or neutrinos.

We commence our discussion towards a formulation of an effective low-energy action for this type of stringy interactions, including the dynamics of the vector field itself, by first reviewing briefly the underlying formalism of the world-sheet deformation that describes the (impulse) interaction of an open string state with a recoiling D-particle [12]. The pertinent vertex operator on the world-sheet boundary $\partial\Sigma$, within the world-sheet σ-model reads:

$$\mathcal{V}^{\rm impulse}_{\rm recoil} = \int_{\partial\Sigma} u_i X^0 \Theta_\epsilon(X^0)\partial_n X^i \ , \tag{4}$$

where X^0 is the target-space time coordinate, obeying Neumann boundary conditions, and X^i ($i = 1, 2, 3$) are the spatial coordinates on the brane, obeying Dirichlet boundary conditions. The notation ∂_n denotes normal derivatives on the world-sheet. By u_i we denote the recoil relativistic three-velocity of the D-particle ($u_i = \gamma v_i = \gamma g_{\rm s}\Delta k_i/M_{\rm s}$, with $\gamma = (1 - \vec{v}^2)^{-1/2}$ the corresponding Lorentz factor, and Δk_i the momentum transfer of the stringy-matter excitation). The operator

$$\Theta_\epsilon(X^0) = -i\int_{-\infty}^{\infty} \frac{d\omega}{\omega - i\epsilon} e^{i\omega X^0} \quad \text{with} \quad \epsilon \to 0^+ \ , \tag{5}$$

is a regularised operator. It can be shown [12] that for $\epsilon^{-2} = 2\ln(A/a^2)$, where A/a^2 is the area of the world-sheet in units of a world-sheet Ultra-Violet (UV) cut-off length scale a, the vertex operator in Eq. (4) satisfies an appropriate logarithmic conformal field theory. The operator has an anomalous dimension $-\epsilon^2/2 \to 0^+$ in the Infra-Red (IR) fixed point $A/a^2 \to \infty$, where we shall work from now on [5].

The impulse operator in Eq. (4) is written in the co-moving frame of the recoil D-particle. In the frame of an observer on the D3-brane, the covariantised form of this expression should be used instead [5]. We can write this as a total world-sheet derivative in the following way:

$$\int_\Sigma \partial_a \left(u_\mu u_\nu X^\nu \Theta_\epsilon(u_\rho X^\rho)\partial^a X^\mu\right) \quad \text{with} \quad \epsilon \to 0^+ \ , \tag{6}$$

where u_μ is a four-velocity vector satisfying:

$$u_\mu u^\mu = -1 \ , \tag{7}$$

and $a = 1, 2$ is a world-sheet index. Note that above, using the Neumann boundary conditions, $\partial_n X^0 = 0$, for the temporal coordinate X^0, thereby implying an ambiguity in Eq. (4) when written as a total world-sheet derivative, allowed us to write the covariant form Eq. (6).

As discussed in detail in Ref. [5], where we refer the interested reader for details, the bulk operator Eq. (6) is conformal on the world-sheet in the sense of having conformal dimension two. However, the target-space metric deformation implied by the vertex Eq. (6) does not connect smoothly with the flat Minkowski space-time before the capture of the open string by the D-particle. Indeed, by expanding the derivative in Eq. (6) and ignoring terms of the form $\partial_\alpha\partial^\alpha X^\mu$, that can be eliminated on-shell in the world-sheet theory, we recognise a contribution similar to the free sigma-model action propagating in a curved background $g_{\mu\nu}$, namely $\int_\Sigma g_{\mu\nu}\partial_a X^\mu \partial^a X^\nu$. In this sense, there is a target-space deformation due to the D-particle recoil:

$$\delta g_{\mu\nu} = u_\mu u_\nu \Theta_{\epsilon\to 0^+}(u_\rho X^\rho). \tag{8}$$

To ensure a smooth connection with Minkowski space-time before the collision/capture, one can couple to this deformed σ-model a linear dilaton background term of the form [5]:

$$\int_\Sigma d^2\xi \phi R^{(2)} \ , \quad \phi = u_\mu X^\mu \ , \tag{9}$$

where $d^2\xi$ is the covariant measure over the curved world-sheet of curvature $R^{(2)}$. Coupling the vertex operator Eq. (6) to e^ϕ factors does not affect the conformal invariance, since the conformal dimension of e^ϕ in the presence of the linear-dilaton background Eq. (9) is $u_\mu(u^\mu - u^\mu) = 0$. Hence the complete vertex operator for recoil, compatible with conformal invariance at the IR fixed-point $\epsilon \to 0^+$, turns out to be [5]:

$$\mathcal{V}_{\rm recoil} = \int_\Sigma e^\phi \partial_a \left(u_\mu u_\nu X^\nu \Theta_{\epsilon\to 0^+}(u_\rho X^\rho) \partial^a X^\mu \right) \ . \tag{10}$$

Acting with the derivative in the above formula, we can see that the deformed target-space metric has the form

$$\delta g_{\mu\nu} = e^\phi u_\mu u_\nu \Theta_{\epsilon\to 0^+}(u_\rho X^\rho). \tag{11}$$

However, to ensure a smooth connection with the flat metric at the origin of the boosted time, $u_\rho X^\rho = 0$, the above metric deformation Eq. (11) must also contain extra dilaton terms, such that the induced metric is of the form:

$$\begin{aligned} g_{\mu\nu}^{\rm neutral\ matter} &= \eta_{\mu\nu} - \left(e^\phi - e^{-\phi}\right) u_\mu u_\nu \ , \\ \phi &= u_\mu X^\mu \ . \end{aligned} \tag{12}$$

Note that the e^ϕ correction corresponds to an operator on the world-sheet of the string with *zero* conformal dimension, $-u_\mu(-u^\mu - u^\mu) + 2 = 2(u_\mu u^\mu + 1) = 0$, due to Eq. (7). This latter deformation leads to *departure from criticality* of the associated σ-model and thus Liouville dressing is required [13].

To this end [5], we first notice that the linear dilaton implies a sub-critical string with $Q^2 = u_\mu u^\mu = -1 < 0$. This can become conformal if one uses a *space-like* Liouville mode [13] ρ to "dress" the above-mentioned metric deformations by multiplication with exponential operators $e^{\alpha_i \rho}$ (with $i = 1, 2$); α_i denote the Liouville "anomalous" dimensions. In the presence of the world-sheet background charges in the (ρ, X^μ) extended target space time of the Liouville-dressed world-sheet theory, induce world-sheet curvature terms of the form

$$\int_\Sigma d^2\xi (u_\mu X^\mu + \rho) R^{(2)}$$

in the σ-model action. The conformal dimension of these operators is $\alpha_i(\alpha_i + 1)$ for each $i = 1, 2$. Restoration of conformal invariance requires that the total conformal dimension of the Liouville-dressed deformations is (1,1) in the (holomorphic, anti-holomorphic) world-sheet sectors. It is straightforward then to observe that the following dressed operators are conformal, amounting to the choices $\alpha_i = \pm|Q| = \pm 1$ in the respective Liouville anomalous dimensions,

$$\mathcal{V}^{\lambda\nu}_{\rm (dressed)} u_\lambda u_\nu = \left(e^{-u_\mu X^\mu - \rho} - e^{u_\mu X^\mu + \rho} \right) \partial X^\lambda \overline{\partial} X^\nu u_\lambda u_\nu \ . \tag{13}$$

These imply a dressed target-space-time metric in the extended space-time (ρ, X) of the form:

$$\begin{aligned} g_{\mu\nu}^{\rm neutral\ matter,\ dressed}(\rho, X) &= \eta_{\mu\nu} + \left(e^{\Phi(\rho,X)} - e^{-\Phi(\rho,X)} \right) u_\mu u_\nu \ , \\ g_{\rho\mu} &= 0 \ , \\ g_{\rho\rho} &= +1 \ , \\ \Phi(\rho, X) &= -u_\mu X^\mu - \rho \ . \end{aligned} \tag{14}$$

The extra space-like Liouville mode may thus be given the physical interpretation of a bulk spatial dimension, in which recoil of the defects does not take place. Here, our brane space-time is located at, say, $\rho = 0$. Note that this is just one example of a consistent conformal theory. In general, one may consider non-trivial σ-model metrics $G_{\mu\nu}$, in which case the associated dilatons will have a more complicated space-time dependence. In this respect, one can discuss the effects of D-foam recoil in realistic Friedman-Lemaître-Robertson-Walker (FLRW) backgrounds, which constitutes the subject of the present paper.

With the above considerations in mind, we take from now on the following form of the deformed target-space metric:

$$\begin{aligned} g_{\mu\nu}^{\rm neutral\ matter} &= G_{\mu\nu} + (e^\phi - e^{-\phi}) u_\mu u_\nu \quad \text{with} \quad \mu, \nu = 0, \dots 4 \ , \\ \phi &= \Phi(0, X^\mu) \quad \text{with} \quad \mu, \nu = 0, \dots 4 \ , \end{aligned} \tag{15}$$

due to D-particle recoil during their interaction with neutral matter. Thus, we assume a three(spatial)- dimensional (D3-)brane universe from now on for brevity (the latter may be obtained by appropriate compactification of higher dimensional branes), ignoring bulk physics as far as the recoil of D-particles is concerned, which we assume to be dominant only on the brane world.

Finally, we remark that the recoil velocity field u_μ is elevated to a dynamical one in this approach, as being part of a gauge background field in the D-particle recoil, which obeys non-trivial dynamics [5]. This can be seen in two equivalent ways. In the first one, we use the fact that, upon T-duality (which exchanges Neumann and Dirichlet boundary conditions and is assumed to be an exact symmetry of the underlying string theory), the (non-covariantised) vertex operator of recoil Eq. (4) is related to the background gauge potential deformation [12]:

$$A_i = u_i X^0 \Theta(X^0) \ , \tag{16}$$

assuming [12], without loss of generality, a time axial gauge $A_0 = 0$. This background corresponds for $X^0 > 0$ (after the impulse) to a constant "electric"-type field $E_i \equiv F_{0i} = u_i$, where $F_{\mu\nu} = \partial_\mu A_\nu$ is the Maxwell field strength of the gauge field. This vector field describes the average effects of the interaction of neutral matter with the background of the recoiling D-particle and should not be confused with the electromagnetic field of the standard model; it is a new degree of freedom associated with the back reaction of the D-matter onto space time.

Alternatively, without invoking T-duality, one may use the world-sheet version of Stokes' theorem, to write down the boundary recoil deformation Eq. (4) (before coupling to the dilaton/Liouville) as a bulk deformation [14]:

$$\begin{aligned} \mathcal{V}^{\rm impulse}_{\rm recoil} &= \frac{1}{2\pi\alpha'}\int_\Sigma d^2\xi\, \varepsilon_{\alpha\beta}\partial^\beta\left(\left[u_i X^0\right]\Theta_\epsilon\left(X^0\right)\partial^\alpha X^i\right) \\ &= \frac{1}{4\pi\alpha'}\int_\Sigma d^2\xi\,(2u_i)\,\varepsilon_{\alpha\beta}\partial^\beta X^0\left[\Theta_\epsilon\left(X^0\right) + X^0\delta_\epsilon\left(X^0\right)\right]\partial^\alpha X^i \ , \end{aligned} \tag{17}$$

where $\varepsilon_{\alpha\beta}$ is the world-sheet Levi-Civita antisymmetric symbol and $\delta_\epsilon(X^0)$ is an ϵ-regularised δ-function. For relatively large times after the impact at $X^0 = 0$ (which we assume here for our phenomenological purposes), this is equivalent to a deformation describing an open string propagating in an antisymmetric $B_{\mu\nu}$-background corresponding to an external constant in target-space "electric" field,

$$B_{0i} = F_{0i} \sim u_i \ \ \text{and} \ \ B_{ij} = 0 \ \ \text{for} \ \ X^0 > 0 \ , \tag{18}$$

where the $X^0\delta(X^0)$ terms in the argument of the electric field yield vanishing contributions in the large time limit, and hence are ignored from now on. In this approach, we note that the presence of the B-field leads to mixed-type boundary conditions for open strings on the boundary $\partial\Sigma$ of world-sheet surfaces with the topology of a disc; we consider here for concreteness and relevance to our discussion below, that:

$$g_{\mu\nu}\partial_n X^\nu + B_{\mu\nu}\partial_\tau X^\nu|_{\partial\Sigma} = 0 \ , \tag{19}$$

where $g_{\mu\nu}$ denotes the metric of target space-time. Formally, the operator Eq. (17) is conformal on the world-sheet, in the limit $\epsilon \to 0^+$ [2] in flat space-times. The generalisation to curved space, and in particular to FLRW backgrounds, will be discussed in subsequent sections.

This conformal gauge field background is assumed to exist together with the metric and dilaton backgrounds, discussed previously. The dynamics of the gauge field describes physics in the so-called open string channel, whilst the dilaton and metric backgrounds describe the effects of the recoiling D-particles in the closed string channel. The above considerations summarise therefore the features of the space-time foam model which motivate the inclusion of vector and scalar fields together with gravitational tensor fields.

The reader should notice a formal similarity of the metric Eq. (15) with the corresponding one in TeVeS models [1], including, in addition to the graviton tensor field $g_{\mu\nu}$, also scalar (dilaton ϕ) and vector (recoil velocity-related gauge A_μ) fields. However, in our case, the interpretation is entirely different from the TeVeS models, and moreover, given that we are dealing with ordinary (from a phenomenological point of view) supersymmetric strings, there are natural candidates for dark matter in our models (the lightest (stable) supersymmetric partners to the standard model excitations), co-existing with the scalar and vector structures. Thus, our models are not meant to provide alternative

[2] Notice that to express the gauge deformation in the frame of a D3-observer, it is not sufficient to replace $X^0 \to u_\mu X^\mu$, in the argument of the $\Theta_{\epsilon\to 0^+}(X^0)$, but also to transform the recoil three-velocity vector u_i. However, for our purposes this is not necessary.

to dark matter scenarios. Nevertheless the existence of dilaton and vector fields can affect certain cosmological features of the string universe, which shall be explored in this article.

In a cosmological context, as already mentioned, one needs to reconsider the conformal invariance conditions for metrics of FLRW type that depend on the cosmic time. These conditions are equivalent to equations of motion for the various target-space fields that are derived from a low-energy string effective action. We next proceed therefore to discuss the effective low-energy action, which can describe in general quantum fluctuations of the tensor, scalar and vector backgrounds and in subsequent sections we shall discuss background solutions to the respective equations of motion and perturbations around them. The background solution we shall deal with in this work is characterised by a constant dilation but non trivial FLRW metrics and recoil vector fields adapted to this geometry, which will generalise the flat space solution (16) mentioned above.

III. LOW-ENERGY EFFECTIVE LAGRANGIAN FOR D-FOAM MODELS: A PROPOSAL

Here, we shall generalise the arguments of the previous section to discuss more general background configurations for the dilaton, graviton and gauge fields, in a way consistent with the conformal invariance of the world-sheet of the string. We would like to search for cosmological backgrounds to be applied in our cosmological tests of the model. In particular, we will investigate the role of vector fields to galaxy growth. We shall construct first an effective low-energy field theory for a graviton, dilaton and vector field background in the context of the model described above and find configurations for the background fields that are consistent with the conformal invariance conditions of the σ-model theory, generalising the analysis of Ref. [5]. Then, we shall analyse the resulting modifications of gravity in a cosmological context, in particular we will study their effects in galactic growth and structure formation. The requirement of conformal invariance of the string theory on the world-sheet is equivalent to the requirement of the background fields satisfying equations of motion arising from the target space-time effective action. As the vector-field part of the action used to describe open strings ending on D-branes is well-known and can be expressed in a closed form to all orders in the string Regge slope α' (DBI action) [15], we are able to start from it and then study its modifications to include the description of gravity in the bulk and on the brane.

A. Dirac-Born-Infeld action on the 3-brane worlds

We commence our analysis from the gauge field effective Lagrangian in four-dimensional flat space-time, representing the case of propagating open strings in gauge-field backgrounds, with their ends attached on a D3-brane. The gauge field backgrounds and the open strings are not propagating in the bulk. In the open string channel, it is known that such a Lagrangian can be obtained to all orders in the α' expansion, and has the DBI form [15]:

$$\mathcal{L}_{\rm DBI} = -\frac{1}{g_{\rm s}} T_3 \sqrt{\det{}_4\left(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}\right)}\ , \tag{20}$$

where T_3 is the three-dimensional brane tension and $F_{\mu\nu}(A)$ is the (Abelian) field strength. It can be readily shown that a constant electric field strength background, as is the case of the recoil-velocity gauge field for times after the impulse, $X^0 > 0$, is a consistent solution of the gauge field equations of motion derived from this action, and thus a consistent (world-sheet conformal invariant) configuration for the string.

In the presence of non-trivial dilatons, the string coupling $g_{\rm s}$ becomes $g_{\rm s0}e^{\phi}$ (where $g_{\rm s0} = e^{<\phi>}$ is a space-time constant). Moreover, in non-Minkowski backgrounds, the tensor $\eta_{\mu\nu}$ should be replaced by the background metric $g_{\mu\nu}$ and thus the DBI Lagrangian Eq. (20) acquires the form:

$$\mathcal{L}_{\rm DBI} = -\frac{1}{g_{s0}} T_3 e^{-\phi} \sqrt{\det{}_4\left(g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}\right)} + \dots\ , \tag{21}$$

where the $\dots$ include other terms, which are non-trivial in the case where the field strength $F_{\mu\nu}$ is non-constant, and also include curvature terms that give the graviton field dynamics. In the closed string channel, of course, the latter terms are just the Einstein-scalar curvature terms to lowest order in derivatives for the gravitational sector. However, an interesting question arises as to whether one may discover curvature terms coupled directly to the DBI action, as a result of induced metrics in the open string channel. It is the point of this section to attempt to address such a question.

First of all we remark that the four-dimensional DBI action can be expanded in derivatives, as appropriate for a low-energy approximation, compared to the string scale $M_{\rm s} = 1/\sqrt{\alpha'}$, as follows [15]:

$$\det{}_4\left(g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}\right) = \det{}_4 g\left[1 + (2\pi\alpha')^2 I_1 - (2\pi\alpha')^4 I_2^2\right]\ , \tag{22}$$

where

$$I_1 = \frac{1}{2} g^{\mu\lambda} g^{\nu\rho} F_{\mu\nu} F_{\lambda\rho} \ , \quad I_2 = -\frac{1}{4} \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho} \ . \tag{23}$$

Upon replacement in the Lagrangian Eq. (21), one obtains:

$$\mathcal{L}_{\rm DBI} = -\sqrt{\det{}_4 g} \left[e^{-\phi} \frac{T_3}{g_{\rm s0}} + \frac{1}{2}(2\pi\alpha')^2 e^{-\phi} \frac{T_3}{g_{\rm s0}}\, I_1 - \frac{1}{2}(2\pi\alpha')^4 e^{-\phi} \frac{T_3}{g_{\rm s0}}\, I_2^2 + \dots + \mathcal{L}_{\rm G} \right] \ , \tag{24}$$

where the ... indicate higher derivative terms, and $\mathcal{L}_{\rm G}$ indicates closed-string sector gravity contributions. Since gravity is allowed to propagate in the bulk, the four-dimensional form of the gravitational terms can be obtained from a projection onto the D3-brane of such terms. In the σ-model frame the latter are of the form:

$$\mathcal{L}_{\rm G} = \sqrt{\det{}_4 g}\, \frac{V^{(6)}}{g_{\rm s0}^2 M_{\rm s}^2} e^{-2\phi} \left(R(g) - \Lambda_2 + 4\partial_\mu \phi \partial^\mu \phi + \dots \right) \ , \tag{25}$$

where $V^{(6)}$ is a volume element of extra (compactified) dimensions (in units of $\sqrt{\alpha'}$), Λ_2 is an effective cosmological constant term, which may arise from compactification, and ϕ is a four-dimensional dilaton kinetic term. The signature of Λ_2 depends on the details of the theory. We define the four-dimensional bulk-induced gravitational constant κ_0 as:

$$\frac{1}{\kappa_0} = \frac{V^{(6)}}{g_{\rm s0}^2 M_{\rm s}^2} \ . \tag{26}$$

Notice that in this model there are two contributions to the dilaton-induced dark energy: one from the open string sector, as a result of the

$$e^{-\phi} \frac{T_3}{g_{\rm s0}} \equiv \Lambda_1(\phi) \ , \tag{27}$$

terms in Eq. (24), and another one from

$$\frac{V^{(6)}}{g_{\rm s0}^2 M_{\rm s}^2} e^{-2\phi} \Lambda_2 \equiv \tilde{\Lambda}(\phi) \ , \tag{28}$$

terms. Combining Eqs. (24), (25), the overall dark energy contributions would then come from

$$\Lambda_{\rm total\ dark\ energy} = \Lambda_1(\phi) + \tilde{\Lambda}(\phi) \ . \tag{29}$$

Since in our approach, Λ_1 is manifestly of fixed sign (positive, for positive tension D3-branes $T_3 > 0$), the overall sign of the induced four-dimensional dark energy $\Lambda_{\rm total\ dark\ energy}$ depends on the relative strength of its constituents. On noting the different scaling of the two types of vacuum energy contributions $\Lambda_1, \tilde{\Lambda}$ with the string coupling $g_{s0} e^{\phi}$, we observe that it is possible to have a positive dark energy on the four-dimensional world, for positive $\Lambda_1(\phi) + \tilde{\Lambda}(\phi)$, with the Λ_1 contribution being sub-dominant for weak string couplings at late times. This is the assumption we shall make when analysing the consequences of the Eq. (24) in cosmic structure (galaxy) formation. For other phases of the brane universe, the sign of $\tilde{\Lambda}(\phi)$ may change. In fact, in the microscopic models of D-foam [6, 7] we are considering here, this sign depends on the ratio of the densities of nearby-to-far-away bulk D-particle defects, with respect to the brane universe. Indeed, the attractive flux forces between brane and bulk defects moving in directions transverse to the brane world, have specific dependence on the relative distance, which are such that for nearby (far away) bulk D-particles (lying at distances shorter (longer) than the string length from the brane universe) there are negative (positive) energy contributions to the brane vacuum potential energy. We shall come back to this important point later on in our article.

The effective actions in Eqs. (24) and (25) do not give the complete action that couples dynamical gravity to the recoil vector field; some further modifications need to be done to the Dirac-Born-Infeld part, which we shall now discuss.

B. Space-time curvature correction terms in the Dirac-Born-Infeld action

We first notice that, in addition to the closed-string sector gravitational field, the effective action of D-foam may contain induced target-space curvature contributions, that couple to the open-string sector, in particular to the DBI

Lagrangian Eq. (21). Such a situation is known to characterise simple examples of Dp-branes (p stands for the dimensionality of the D-brane), fluctuating in the bulk space [16]. In fact, the authors of Ref. [16] represented the essential features of such fluctuating branes by considering toy examples of world-sheet actions for a Dp-brane with tachyonic-type deformations of the form

$$S_\sigma = \frac{1}{4\pi\alpha'}\int_\Sigma \partial X^M \bar{\partial} X_M + \frac{c}{4\pi\alpha'}\int_{\partial\Sigma}\left(X^i - Y^i(X^\mu)\right)^2 \ ; \tag{30}$$

where X^M with $M = 0, \ldots, D-1$, are the string coordinates. In this notation, the index μ takes values from 0 to p and corresponds to the $p+1$ coordinates that obey Neumann boundary conditions, while the rest obey Dirichlet boundary conditions; the latter are denoted by X^i. The limit $c \to \infty$, which corresponds to a conformally invariant point of the theory, ensures that the end-points of the strings are confined to move on the hyper-surface defined by Y^i. In this limit, a space-time effective action for the Y^is can be identified with the partition functional of c and Y^i:

$$S(Y^i) = \lim_{c\to\infty} Z\left(c, Y^i\right) = \int dX^i dX^\mu e^{-S_\sigma} \, . \tag{31}$$

The authors of Ref. [16] split the string fields in the usual way in a zero mode plus perturbations,

$$X^M = x^M + \tilde{X}^M \, , \tag{32}$$

and Taylor expanded $Y^i(X)$ around $Y^i \equiv Y^i(x)$. They then integrated out of the partition function the contribution of $\tilde{X}$, and, after expanding to orders in $(x^i - Y^i)K^i_{\mu\nu}$, with $K^i_{\mu\nu} = \partial_\mu\partial_\nu Y^i$ being the extrinsic curvature, they integrated the quadratic and quartic terms and arrived at

$$Z(c, Y) = Z(c)\int dx^\mu \sqrt{\det G}\left[1 - \alpha'\zeta(2)R(G) + \mathcal{O}(\alpha'^2)\right] \ , \tag{33}$$

where $\zeta(2) = \pi^2/6$ and $G_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu Y^i \partial_\nu Y^i$.

In the flat-brane limit of Ref. [16], the induced metric G is the only gravitational contribution. One, however, may generalise the background G to an arbitrary metric, g, which should then be a consistent solution to the classical equations of motion for the gravitational field, obtained from this modified gravity action. The presence of the curvature corrections in the partition function Eq. (33) of the open string, imply a target-space effective action of the form

$$S = -\frac{T_3}{g_\mathrm{s}}\int d^4x \sqrt{-\det g}\left[1 - \alpha'\frac{\pi^2}{6}R + \mathcal{O}\left(\alpha'^2\right)\right] \ , \tag{34}$$

which is just the ordinary Einstein-gravity theory (to lowest order in derivatives), in the presence of a positive cosmological constant T_3/g_s. From this point of view, the gravitational (Planck) constant for this theory is $\kappa = g_\mathrm{s}/(\alpha' T_3)$. However, the reader should also notice that there are curvature contributions to the effective action, coming from the closed string sector, of the form Eq. (25). In the case of constant dilatons (as is the case of the model of Ref. [16]), this will lead to additional contributions to the effective gravitational constant in four-dimensions, coming from the closed string sector, while in the case of time-dependent dilatons, of interest to us here, one would obtain field-varying gravitational couplings. We shall come back to this point in the next section.

For the moment we note that coupling a U(1) gauge field background to the model, Eq. (30), corresponds to adding the following boundary term to the action S_Σ:

$$\frac{1}{\sqrt{2\pi\alpha'}}\oint_{\partial\Sigma} A_\mu(X)\partial_\tau X^\nu = \frac{1}{2\sqrt{2\pi\alpha'}}\int_\Sigma \varepsilon_{\alpha\beta}F_{\mu\nu}\partial^\alpha X^\mu \partial^\beta X^\nu \ , \tag{35}$$

where we have used Stokes's theorem, and the indices α, β denote world-sheet indices. The $\sqrt{2\pi\alpha'}$ comes from dimensional considerations. For constant field strength backgrounds this is formally equivalent to an antisymmetric $B_{\mu\nu}$-tensor background. In general, this is not the case. This addition changes the effective action Eq. (33) in the two following ways.

Firstly, the determinant of the (induced) metric changes as

$$\sqrt{\det g} \to \sqrt{\det(g + 2\pi\alpha' F)} \equiv \sqrt{\det(h)} \, , \tag{36}$$

where we consider a general metric background g, according to our previous discussion, and not only the induced metric G. As we have discussed in the previous sub-section, the determinant given in Eq. (36) is the one that appears in the DBI action in a curved background, Eq. (21). In fact, the tensor $h_{\mu\nu}$ is defined as

$$h_{\mu\nu} \equiv g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu} \ , \tag{37}$$

and its inverse is denoted by $h^{\mu\nu}$, with $h^{\mu\nu}h_{\nu\rho} = \delta^{\mu}_{\ \rho}$. Secondly, as it is well-known from matching with string-amplitude calculations [17], in addition to the curvature terms in Eq. (34), there are contributions involving the gauge field and its target-space derivatives (in the general case of non-constant field-strength backgrounds). In the superstring theories we are interested in (despite the fact that we are only considering the bosonic parts of the pertinent target-space effective actions), it is known that such corrections start from structures with four-derivatives acting on the field strengths, i.e. of the form [18]

$$S_{\rm DBI} = -\frac{T_3}{g_s}\int d^4x \sqrt{\det(h)}\left[1 - \alpha'\frac{\pi^2}{6}R + \mathcal{O}_{\rm curv}(\alpha'^2) + \mathcal{O}\left((\partial\partial F, \partial F \partial F)^2\right)\right] \ , \tag{38}$$

where $\mathcal{O}_{\rm curv}(\alpha'^2)$ indicates terms involving curvature squared terms. All such higher-derivative terms will be ignored in our low-energy considerations.

IV. THE FOUR-DIMENSIONAL INDUCED EFFECTIVE ACTION

Following the above discussion, we therefore use the four-dimensional effective action (on a D3-brane world)

$$\begin{aligned} S_{\rm eff\ 4dim} \ &= \ \int d^4x \left[-\frac{T_3}{g_{s0}} e^{-\phi}\sqrt{-\det\left(g + 2\pi\alpha' F\right)}\left(1 - \alpha R(g)\right) \right. \\ &\left. - \sqrt{-g}\, e^{-2\phi}\,\frac{1}{\kappa_0}\,\tilde{\Lambda} + \frac{1}{\kappa_0}\sqrt{-g}\, e^{-2\phi}\, R(g) + \mathcal{O}((\partial\phi)^2)\right] \ , \end{aligned} \tag{39}$$

(where $g = \det(g)$) for the gravitational field $g_{\mu\nu}$, the dilaton field ϕ and the gauge field A_μ, with field strength $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ on a D3-brane. Note that κ_0 is defined in Eq. (26), and $\alpha = \alpha'\zeta(2) = \alpha'\pi^2/6$ in the example of Ref. [16], however we wish to keep the discussion general and so we use from now on the positive constant $\alpha > 0$ as a parameter of our model; it has dimensions of length squared. In this respect, we also assumed a non-constant dilaton, thereby writing the string coupling as $g_s = g_{s0}e^{\phi}$, with $g_{s0} < 1$ a (perturbatively) weak constant string coupling. The terms $\mathcal{O}((\partial\phi)^2)$ denote kinetic terms of the dilaton, which shall not play an important role in our analysis in this work, for reasons that will be clarified below; hence we do not need to write them down explicitly.

Using Eq. (22), taking into account that for the background solutions configurations pertinent to the effects of the D-particle foam only "electric-field" type components F_{0i} are non-trivial (*i.e.*, $I_2 = 0$ in Eq. (23)) and expanding the square root in Eq. (39), while keeping up to four-derivative-order terms, we finally arrive at the following form of the four-dimensional effective action on the D3-brane world

$$\begin{aligned} S_{\rm eff\ 4dim} \ &= \ \int d^4x \sqrt{-g}\left[-\frac{T_3}{g_{s0}}\, e^{-\phi} - e^{-2\phi}\,\frac{1}{\kappa_0}\,\tilde{\Lambda} \right. \\ &- \frac{T_3}{4g_{s0}} 2\pi\alpha'\, e^{-\phi} F^{\mu\nu}F_{\mu\nu} + \alpha\frac{T_3}{4g_{s0}} 2\pi\alpha'\, e^{-\phi}F^{\mu\nu}F_{\mu\nu}R(g) \\ &\left. + \left(\alpha\frac{T_3}{g_{s0}}\, e^{-\phi} + \frac{1}{\kappa_0}\, e^{-2\phi}\right) R(g) + \mathcal{O}\left((\partial\phi)^2\right)\right] \ . \end{aligned} \tag{40}$$

In view of the mis-match between open and closed string sectors, we do observe from Eq. (40) that the coefficients of the Einstein scalar curvature term R are not of the usual Brans-Dicke type.

We shall base our subsequent analysis upon Eq. (40). We may proceed by redefining the space-time metric in such a way that the bulk-induced Einstein curvature term $(1/\kappa_0)\, e^{-2\phi}\, R(g)$ acquires a canonically normalised term without dilaton couplings, that is we redefine:

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = e^{-2\phi} g_{\mu\nu} \ , \tag{41}$$

which defines the so-called Einstein-frame metric [19]. In our conventions, such a redefinition implies for our four-dimensional brane world:

$$\tilde{R}(\tilde{g}) = e^{2\phi}\left[R(g) - \frac{6}{\sqrt{-g}}\,\partial_\mu\left(\sqrt{-g}\, g^{\mu\nu}\,\partial_\nu\phi\right) + 6\, g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\right] \ . \tag{42}$$

Upon combining Eq. (41) with a canonical normalisation of the kinetic terms of the gauge fields, by absorbing the factor $(2\pi\alpha' e^{-\phi_0} T_3/g_{s0})^{1/2}$ into a redefinition of the gauge potential A_μ, we finally obtain the following effective action in the Einstein frame $S^E_{\rm eff~4dim}$ (for brevity from now on we omit the tilde notation from the Einstein metric $\tilde{g}_{\mu\nu}$):

$$S^E_{\rm eff~4dim} = \int d^4x \sqrt{-g}\, \Big[-\frac{T_3}{g_{s0}}\, e^{3\phi} - e^{2\phi}\, \frac{1}{\kappa_0}\, \tilde{\Lambda} - \frac{1}{4}\, F^{\mu\nu} F_{\mu\nu} \\ + \alpha \frac{1}{4} F^{\mu\nu} F_{\mu\nu} e^{-2\phi} R(g) + \left(\alpha \frac{T_3}{g_{s0}}\, e^{\phi} + \frac{1}{\kappa_0} \right) R(g) - \frac{3}{2}\, \alpha\, e^{2\phi} g^{\mu\nu}\, \partial_\mu (F_{\alpha\beta}\, F^{\alpha\beta})(\partial_\nu \phi) + \mathcal{O}'\left((\partial\phi)^2 \right) \Big] + S_m, \quad (43)$$

up to terms proportional to the square of dilaton gradients $\partial_\mu\phi$, denoted by $\mathcal{O}'\left((\partial\phi)^2\right)$, which we shall ignore, as we have already mentioned. The reader should have noticed, however, that terms containing one derivative of the dilaton have been kept, since they will yield non-trivial contributions to the dilaton equation of motion even in the case of constant dilaton, we are considering. In Eq. (43) we have added the matter sector action S_m which we do not specify. Notice that for effectively constant dilaton $\phi = \phi_0$ we are interested in here, the gravitational constant (that is the coefficient of the Einstein term) reads

$$\frac{1}{\kappa} \equiv \frac{1}{\kappa_0} + \alpha \frac{T_3}{g_{s0}}\, e^{\phi}\ . \quad (44)$$

This is the action we shall use from now on in order to analyse the effects of the vector perturbations in structure formation in this stringy universe. Notice that, for the sake of brevity, we used above the same symbol F for the field strength of the canonically normalised gauge fields.

Before proceeding to discuss the role of perturbations, let us discuss first the background configurations about which we shall perturb. First we may assume that at early epochs of the universe, which will not be the subject of our study here, there is a de Sitter phase of the brane universe, in which the population of D-particle defects is sufficiently dilute, so that any recoil vector field strength F contributions to the effective action are *negligible*. From Eq. (43) we observe that apart from the term $-(T_3/g_{s0})\, e^{3\phi}$, the rest of the terms in the effective action have the form of a low-energy gravitational action coming from the closed sector of string theories, within graviton, dilaton (ϕ) and tachyon ($T(x)$) backgrounds, the latter one corresponding to initial cosmological instabilities. Such de Sitter phases have been argued to be consistent with world-sheet conformal invariance to all orders in α' Ref. [20]. In fact, the de Sitter phase corresponds to a relatively brief cosmological era, during which the amplitudes of the (classical) tachyon and dilaton fields are proportional to each other, $T(x) + \phi(x) = 0$. Once such a field alignment is lost, due to a decay in time of the tachyon instability, the universe exits dynamically the de Sitter phase. It is important to notice that in the solution of Ref. [20], the dilaton in the Einstein frame also assumes in such a phase a logarithmic Robertson-Walker-frame time dependence, $\phi = \phi_0 {\rm ln} t$, $\phi_0 < 0$. This implies that the afore-mentioned open-string term $-(T_3/g_{s0})\, e^{3\phi}$ is negligible compared to the rest of the terms in the effective action Eq. (43) and thus it can be approximated by the one derived from closed strings. This justifies the existence of an approximately de Sitter phase in our context.

At late eras, for instance those corresponding to red-shifts around $z \simeq 2$, relevant for structure formation which we are interested in, the D-particle populations, especially near galactic centres, are significant, thereby leading to the presence of recoil vector field F-contributions to the effective action. In such eras, the background configuration for the D-particle recoil operators which satisfies the (logarithmic) conformal algebra on the world-sheet of the string [21], and is therefore a consistent solution of the equations of motion of the target-space effective action, involves approximately constant dilatons and spatially flat Robertson-Walker metric backgrounds in the Einstein frame.

This, in conjunction with Eq. (15), gives us that

$$\phi = \phi_0 = {\rm const}\ , \quad g_{00} = -1 + 2\sinh(\phi_0) u_0^2\ , \quad g_{ij} = a^2(t)(\delta_{ij} + 2\sinh(\phi_0) u_i u_j)\ . \quad (45)$$

However, in what follows we shall redefine the coordinates in such a way so that the standard FLRW metric is used as our space-time background in order to make contact with observations.

The configuration for the D-particle recoil operator in such backgrounds reads [3]:

$$V = \int_{\partial\Sigma} g_{ij}(t) y^j(t) \Theta(t - t_{\rm impact}) \partial_n X^i\ , \quad (46)$$

[3] For completeness we note at this stage that, even if we worked with the metric Eq. (45), the factors depending on $\sinh(\phi_0)\, u_i^2$ can be ignored below, because they give rise to terms cubic in u_i or higher. However, by performing the above-mentioned coordinate transformations, we make sure that no extra factors involving the dilaton ϕ_0 would hamper the comparison with the data.

where ∂_n denotes the normal derivative on the world-sheet boundary $\partial\Sigma$, t is the cosmic time, $g_{ij}(t) = a^2(t)$ is the spatial part of the FLRW metric and

$$y^i(t) = \frac{v^i}{1-2p}\left(t\frac{a^2(t_{\rm impact})}{a^2(t)} - t_{\rm impact}\right) + \mathcal{O}(t^{1-4p}) \simeq -\frac{v^i}{1-2p}t_{\rm impact}\ , \qquad \text{with}\ \ t \gg t_{\rm impact}\ , \tag{47}$$

is the D-particle recoil trajectory, with $\vec{v}$ the D-particle recoil three velocity in the co-moving frame and $t_{\rm impact}$ the moment of impact of the matter string with a single D-particle. Let us note that this expression stems from the geodesic trajectories of a recoiling D-particle in the metric background [21].

In our study we consider the interaction of strings with populations of D-particles, in which case $t_{\rm impact}$ is an averaged time, considered in what follows as a free constant. Let us, for convenience, absorb the constant $t_{\rm impact}/(1-2p)$ in the definition of velocities, which also absorbs the normalisation factors $(2\pi\alpha' e^{-\phi_0}T_3/g_{s0})^{1/2}$ employed above in order to normalise the Maxwell terms in Eq. (43). This is equivalent to considering a vector gauge field background with spatial components of the form:

$$A_i(t) = -a^2(t)\,\delta_{ij}v^j = -a(t)\delta_{ij}v^j_{\rm (phys)}\ , \tag{48}$$

where $v^j_{\rm (phys)} = a(t)v^j$, $j = 1,2,3$ is the physical ("local") recoil velocity. The above analysis holds for low three velocities; otherwise, one should replace in Eq. (48) the three velocity by the spatial components of the four-vector of the velocity, $u^i = \gamma v^i$ with $\gamma = 1/\sqrt{1 - v^i v^j g_{ij}}$.

The background Eq. (48) corresponds to a field strength of "electric field" type:

$$F_{0i}(t) = -2\dot{a}(t)a(t)u_i\ . \tag{49}$$

where the over-dot denotes the derivative with respect to cosmic time t.

We remind the reader that in this formalism [12], conformal invariance of the stringy σ-model does not restrict the form of the temporal component of the four-vector field A_μ, apart from the fact that homogeneity and isotropy require it to depend only on the cosmic time, $A_0(t)$. This freedom allows us to covariance the background vector field as follows:

$$A_\mu = -a^2(t)u_\mu = -a(t)\,u^{\rm phys}_\mu\ . \tag{50}$$

In the Roberson-Walker background of Eq. (45), the physical ("local") four velocity $u^{\rm phys}_\mu \equiv a(t)\,u_\mu$ obeys the Minkowski-flat constraint:

$$u^{\rm (phys)}_\mu u^{\rm (phys)}_\nu\,\eta^{\mu\nu} = -|{\rm constant}| = -\frac{|{\rm T}_3|}{{\rm g}_{s0}}2\pi\alpha'{\rm e}^{-\phi_0} < 0\ , \tag{51}$$

from which it follows that the vector field has the following time-like constraint in our Robertson-Walker background:

$$A_\mu A_\nu g^{\mu\nu} = -\frac{|T_3|}{g_{s0}}2\pi\alpha' e^{-\phi_0} < 0\ . \tag{52}$$

Unlike the case of Ref. [1], however, in our model the vector field has non-trivial space-like components, proportional to $a^2(t)u_i$. When we consider perturbations, we shall maintain the above constraint, Eq. (52), which is implemented in a path integral via an appropriate Lagrange multiplier $\lambda(x)$ term in the effective action, namely

$$S_{\rm eff-Lagrange} = \int d^4x\sqrt{-g}\,\lambda(x)\,\left(A_\mu A^\mu + \frac{|T_3|}{g_{s0}}2\pi\alpha' e^{-\phi_0}\right)\ , \tag{53}$$

which notably does not couple to the dilaton field. The above should be added to the right hand side of Eq. (43).

We next remark that, upon averaging over foam populations, one assumes the following Gaussian stochastically fluctuating configurations:

$$\ll u_i \gg = 0 \quad \text{and} \quad \ll u_i u_j \gg = \sigma_0^2\delta_{ij}\ , \tag{54}$$

where $\ll \cdots \gg$ indicates appropriate averages over D-particles populations in the foam. Thus, we observe that only upon averaging over the isotropic foam, we obtain vanishing of the spatial components of the A_μ field and of the "electric field", Eq. (49), namely

$$\ll A_i \gg = 0 \quad \text{and} \quad \ll F_{0i} \gg = 0\ . \tag{55}$$

Nevertheless, in our case caution should be exercised to take these averages only at the very end of the computations, since even powers of u_i yield non-zero population averages. The parameter σ_0^2 is free in our model, albeit small, to be constrained by the data. In general, σ_0^2 is cosmic time dependent since it is a function of the (bulk space) density of the foam, which may vary for different cosmological eras [6, 7]. In general, assuming a no-force condition among the D-particles [7], which characterises D-foam models derived from super-membrane theories [6], implies that the density of foam scales as "dust" with the cosmological scale factor, $a^3(t)$, so that to a good approximation over the time period of structure formation we are interested in here, we may take

$$\sigma_0^2(t) = \frac{\beta}{a^3(t)} \frac{2\pi\alpha' |T_3| e^{-\phi_0}}{g_{s0}} \ , \quad \beta = \text{constant} \ > \ 0 \ . \tag{56}$$

Here the factor $2\pi\alpha' e^{-\phi_0}|T_3|/g_{s0}$ comes from the constants which were previously absorbed into the definitions of the recoil velocities.

Some important remarks are due at this juncture. The above considerations are in agreement with the isotropy and homogeneity of the observed Universe. However, for completeness we should mention that when considering the effects of our model at galactic scales, it is possible that certain inhomogeneities in the density of the D-particles may occur in such a way that D-particle populations dominate the haloes of galaxies, thus playing a role analogous to dark matter. Indeed, due to their localised nature (as contrasted with the space filling D-brane universes), D-particles may also be considered as massive excitations of the vacuum with mass of order $M_{\rm s}/g_{\rm s}$. Since there is a no-force condition among them, such particles are ideal to play the role of super-weak dark matter (termed *D-matter* [22]), and if they are sufficiently light, as is the case of low string scales $M_{\rm s}$ of order TeV, and $g_{\rm s} = \mathcal{O}(1)$, they may have implications at colliders, where they can be produced by the collision of Standard Model particles [7, 22]. However, as discussed in Ref. [7], and reviewed briefly above, in our model of D-brane universes propagating in bulk spaces punctured by D-particle populations, there are contributions to the vacuum energy on our brane world by the bulk D-particles, of mixed signature, that depend on the relative distance of the D-particle from the brane world. Negative (positive) energy contributions are due to bulk D-particles that lie at distances smaller (larger) than the string scale from the brane. As a consequence of such contributions, the density of D-particle populations that are trapped on our brane world is not restricted by the requirement of avoiding over-closure of the Universe and can thus be an arbitrary phenomenological parameter. In this sense, scenarios in which various mass density profiles for D-matter can exist in the haloes of galaxies, playing a role analogous to dark matter, are compatible with current cosmology. When we adapt our model above, Eq. (43), to such a situation, we observe that, in view of our coupling of the vector recoil field to the curvature of space-time, averaging over populations of D-particles in the galactic haloes, will result, at a background level, in extra contributions of the recoil term $\ll e^{-3\phi} F_{\mu\nu} F^{\mu\nu} \gg$ to an "effective" gravitational "constant" , proportional to σ_0, which in inhomogeneous scenarios may depend on the particular galaxy:

$$\frac{1}{\kappa_{\rm eff}} \equiv \frac{1}{8\,\pi\,G_{\rm eff}} = \frac{1}{\kappa} + \frac{\alpha}{4} \ll e^{-3\phi} F_{\mu\nu} F^{\mu\nu} \gg \ , \tag{57}$$

where κ is defined in Eq. (44) and plays the role of the four-dimensional Newton's constant $G_{\rm N}$ in our stringy framework. Such an effective gravitational constant will lead to Einstein equations coupling the matter and gravitational systems with a space and time varying effective $G_{\rm eff}$ which can then affect cosmological considerations. Although such scenarios are worth pursuing further, especially as providing alternatives to conventional dark matter (in view of the absence of any concrete particle physics evidence for dark matter at present), nevertheless in the current discussion we shall restrict ourselves to homogeneous and isotropic densities of D-particles on our brane universe, which satisfy Eq. (56). Our aim in this paper is to investigate the effect of the recoil vector field to the growth of galaxies and not to provide realistic alternatives to dark matter scenarios. Our super-membrane effective field theory has its own dark matter, provided, for instance, by the supersymmetric partners of the low-energy effective action. Nevertheless, we hope to come back to such interesting variants of our model in a future work.

In the next section we proceed to consider perturbations around the background solution, Eq. (45), and discuss their relevance to structure growth. To compare our analysis with that of Ref. [11], on the importance of the vector field for structure growth, we neglect in what follows fluctuations of the dilaton field, assuming them to be suppressed as compared to the gravitational and vector perturbations. This is consistent with the phenomenology of low-energy physics at late eras of the universe, given that such dilaton fluctuations would affect the values of the string coupling e^{ϕ}, and through it of all the coupling constants of the string-inspired effective theory, which would not be acceptable from a particle physics viewpoint.

V. BACKGROUND CONFIGURATION AND EQUATIONS OF MOTION

By varying the effective action given in Eq. (43) with respect to the vector, metric and dilaton fields we obtain respectively [4]:

$$\left[F_{\nu\mu}\left(1-\alpha e^{-2\phi_0}R\right)\right]^{;\nu}+2\lambda(x)A_\mu \;=0\;, \tag{58}$$

$$\begin{aligned}&\left[\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}+\frac{\tilde{\Lambda}\,e^{2\phi_0}}{\kappa_0}+\frac{T_3e^{3\phi_0}}{g_{s0}}-\lambda(x)(A_\alpha\,A^\alpha+\frac{|T_3|}{g_{s0}}2\pi\alpha'\,e^{-\phi_0})\right]\frac{g_{\mu\nu}}{2}\\&+\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right)\left[\frac{1}{\kappa_0}+\alpha\,\frac{T_3}{g_{s0}}\,e^{\phi_0}+\frac{\alpha e^{-2\phi_0}}{4}F^{\alpha\beta}F_{\alpha\beta}\right]-\frac{1}{2}g^{\sigma\lambda}F_{\mu\lambda}F_{\nu\sigma}(1-\alpha e^{-2\phi_0}R)\\&+\frac{\alpha e^{-2\phi_0}}{4}\left\{g_{\mu\nu}\nabla^2\left[F^{\alpha\beta}F_{\alpha\beta}\right]-\nabla_\mu\nabla_\nu\left[F^{\alpha\beta}F_{\alpha\beta}\right]\right\}=T^{\rm m}_{\mu\nu}-\lambda(x)A_\mu\,A_\nu\;,\end{aligned} \tag{59}$$

and

$$-\frac{3}{2}\,\alpha\,e^{2\phi_0}\nabla^2(F_{\alpha\beta}\,F^{\alpha\beta})+3\frac{T_3}{g_{s0}}e^{3\phi_0}+2\frac{\tilde{\Lambda}}{\kappa_0}e^{2\phi_0}-\alpha\left(\frac{T_3}{g_{s0}}\,e^{\phi_0}-\frac{1}{2}F_{\mu\nu}F^{\mu\nu}\,e^{-2\phi_0}\right)R(g)=0\;, \tag{60}$$

where κ_0 is defined in Eq. (26), $T^{\rm m}_{\mu\nu}$ is the matter stress tensor and we recall that $\alpha=\alpha'\zeta(2)=\alpha'\pi^2/6>0$ in the D-brane example of Ref. [16]. Above, we have assumed for simplicity that the dilaton coupling to the matter action is not dominant (or equivalently that the dilaton couples only through exponential couplings of the form $\exp(\gamma_i\,\phi)$, with γ_i constants, to the various matter fields, such that upon considering the corresponding dilaton variations, one obtains terms proportional to the Lagrangians of the various matter-species i that vanish on shell). As our primary purpose here is to investigate the role of the vector perturbations on the structure formation and growth, we believe that such an assumption is reasonable.

Before actually solving the system of equations (58), (59) and (60) for the galactic growth and structure formation epoch of the universe, we consider it as instructive to discuss the emergence of a de Sitter phase in this system, which as we shall argue may indeed characterise *late eras* in the evolution of this stringy universe, thereby making our model consistent with the current cosmological observations indicating the dawn of an accelerating de Sitter evolution era once again in the history of our observable Universe.

A. Condensates of the recoil vector field and late-epoch phases of the D-foam universe

To demonstrate the existence of a de Sitter era we first remark that the dynamics of the system of the recoil vector field is described by a Dirac-Born-Infeld Lagrangian given in Eq. (21). For such lagrangians, the higher order interaction terms among the vector field strength may lead to condensates, in analogy with the gluon condensates of Quantum Chromodynamics. In fact, such an assumption has been made in Ref. [23], for generic Abelian flux fields that characterise D-brane excitations, with Dirac-Born-Infeld world volume actions. The analysis in that work indicated then that under certain plausible assumptions on the dominance of the quantum effects (over classical, thermodynamical) on the formation of the condensates, it is possible to obtain an equation of state for the vector Dirac-Born-Infeld system that resembles that of a de Sitter phase, i.e. a cosmological constant type with $w\simeq-1$.

Let us first review briefly, for instructive purposes, those results. Consider a generic Dirac-Born-Infeld field, with Lagrangian Eq. (20) on a D3-brane world volume. In general, there are two contributions to the vector field strength $F_{\mu\nu}$ condensates and one may come from purely quantum vacuum effects,

$$\langle F_{\mu\nu}\,F^{\mu\nu}\rangle_{\rm vac}=\tilde{\alpha}(t)\;,\quad\langle F_{\mu\nu}\,F^{\star\,\mu\nu}\rangle_{\rm vac}=\tilde{\beta}(t)\;, \tag{61}$$

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[4] We set to zero the terms involving derivatives of the dilaton only at the end of the variations, since the latter is assumed constant ϕ_0 and absorbing the *constant* factors $e^{-\phi_0/2}$ into a normalisation of the gauge potential A_μ, so that in this case the kinetic term of the recoil gauge field assumes a canonically normalised Maxwell form. The constraint Eq. (52), is then modified appropriately, however the dilaton equation remains unaffected, since the dilaton does not couple to the constraint term, as already mentioned.

where $F^\star$ indicates the dual tensor, and the condensates may in general depend on time, but are constant on spatial hyper-surfaces. The isometry structure of the spatial hyper-surfaces lead the authors of Ref. [23] to further assume the following, which we also adopt here:

$$\langle F_{0\,\nu}\, F_0{}^{\nu}\rangle_{\rm vac} = \frac{\alpha_{\rm t}(t)}{4}\, g_{00}\ , \quad \langle F_{i\,0}\, F^{0}_{\ j}\rangle_{\rm vac} = \frac{\alpha_{\rm s}(t)}{4}\, g_{ij}\ , \tag{62}$$

where i,j spatial indices on the three-dimensional volume of the D3-brane and we have the relations $\alpha_{\rm t} + 3\alpha_{\rm s} = 4\tilde{\alpha}$. In addition to the quantum effects, one also has classical thermodynamical effects on the energy density and pressure of the Dirac-Born-Infeld fluid, which are obtained by averaging over the spatial volume. If one decomposes the field strength into "electric-field", $F_{0i} \equiv E_i$, and "magnetic-field" $B_i = \frac{1}{2}\epsilon_{ijk}\, F^{jk}$ components, one may specify these classical effects contributions on the condensates [23]:

$$\langle F_{0\,\nu}\, F_0{}^{\nu}\rangle = \langle \sum_{i=1}^{3} E_i^2\rangle\ , \quad \langle F_{i\,0}\, F^{0}_{\ j}\rangle = -\langle E_i\, E_j\rangle + 2\langle B_i\, B_j\rangle\ . \tag{63}$$

On making the further (natural) assumption [23] $\langle E_i\, E_j\rangle = \langle B_i\, B_j\rangle = \mathcal{C}\, g_{ij}/3$, we observe that the total contribution of both classical and quantum effects to the vacuum condensates can then be expressed by the relations:

$$\alpha_{\rm t} = \tilde{\alpha} - 4\mathcal{C}\ , \quad \alpha_{\rm s} = \tilde{\alpha} + \frac{4}{3}\,\mathcal{C}\ . \tag{64}$$

Computing the stress tensor of the Dirac-Born-Infeld fluid, Eq. (20), for the case of the total condensates one arrives at the following expressions for energy density $\rho^{\rm DBI}$ and pressure $p^{\rm DBI}$ [23]:

$$\rho^{\rm DBI} = \frac{\lambda}{2}\left(\frac{1+\frac{\tilde{\alpha}}{2}-\frac{\alpha_{\rm t}}{4}}{\sqrt{1+\frac{\tilde{\alpha}}{2}-\frac{\tilde{\beta}^2}{16}}}\right)\ , \quad p^{\rm DBI} = -\frac{\lambda}{2}\left(\frac{1+\frac{\tilde{\alpha}}{2}-\frac{\alpha_{\rm s}}{4}}{\sqrt{1+\frac{\tilde{\alpha}}{2}-\frac{\tilde{\beta}^2}{16}}}\right)\ , \quad \lambda \equiv \frac{T_3}{g_{\rm s}}\ . \tag{65}$$

Notice that in our case (Eq. (20)) we did not subtract the determinant of the metric $\sqrt{-g}$ in our definition of the action, as this plays the role of a cosmological constant contribution, which we keep. This explains the difference between the right-hand-side of the above expressions and the corresponding ones in Ref. [23], by constant terms of the form $\lambda/2$ and $-\lambda/2$ for $\rho^{\rm DBI}$ and $p^{\rm DBI}$, respectively.

The quantum condensate $\tilde{\alpha}$ and β have not been specified. The only restrictions come from the positivity of the corresponding quantities inside the square root in the Dirac-Born-Infeld action, *e.g.* in the denominators of Eq. (65), which imply the following condition between the various quantum condensates:

$$\frac{\tilde{\alpha}}{2} > -1 + \frac{\tilde{\beta}^2}{16}\ . \tag{66}$$

From Eq. (64), it becomes immediately apparent that on the one hand, if quantum effects are the dominant ones, with $\tilde{\alpha} \gg \mathcal{C}$, then $\alpha_{\rm t} \simeq \alpha_{\rm s} \simeq \tilde{\alpha}$, and hence from Eq. (65) we obtain an equation of state of cosmological constant (de Sitter vacuum) type, namely $w^{\rm DBI} \simeq -1$ independently of the exact form of the quantum condensate (assuming of course it exists, which is a question that probably cannot be addressed in a generic manner, as it requires specific properties of the brane action). On the other hand, as demonstrated in Ref. [23], when classical effects dominate, with $\mathcal{C} \gg \tilde{\alpha}$, then Eq. (64) implies $\alpha_t = -3\alpha_s \simeq -4\mathcal{C}$ and thus Eq. (65) leads to an ordinary relativistic fluid with positive energy density and pressure, $\rho^{\rm DBI} \simeq (\lambda/2)\mathcal{C} > 0$ and $p^{\rm DBI} = (1/3)\rho^{\rm DBI} \simeq (\lambda/2)\mathcal{C} > 0$.

Hence, the effects of the classical (thermodynamical) contributions to the condensates are to cause the equation of state of the Dirac-Born-Infeld fluid to deviate from the value $w_{\rm DBI} \simeq -1$ by an amount that depends on the relative strength of those effect with respect to the quantum ones.

In the context of our study here, the Dirac-Born-Infeld vector field has a microscopic origin, as describing the dynamics of the recoil degrees of freedom of the D-particles under their interaction with electrically neutral matter. In contrast to the case of Ref. [23], here we have the additional couplings of the space-time curvature to the Dirac-Born-Infeld action, Eq. (21), whose effects have been ignored in Ref. [23]. Nevertheless, as we shall argue, for sufficiently small values of the condensate, one can still argue on a late de Sitter era, provided again the quantum vacuum effects dominate the formation of the condensate.

At this point we would like to emphasise two important remarks. Firstly, in our case the *classical* effects on the condensates are just the stochastic effects obtained upon averaging over statistical populations of D-particles, Eqs. (54), (55) and (56). However, the *quantum* effects have a genuine quantum nature due to the quantum-gravitational fluctuations of the D-particle defects in the absence of any matter. The latter are also described by the stretching of

virtual strings between the D-particle and the brane world but they are non-perturbative, and can only be computed in the strongly coupled regime of the target space action of the D-particle, therefore only within the context of M-theory and this is not yet known. Nevertheless, such effects do exist and in what follows we shall adopt a phenomenological approach and simply assume (as in the case of Ref. [23]) that they can condense, when the D-particle populations are sufficiently dense on the brane world, and can lead to the quantum Dirac-Born-Infeld type condensates, Eqs. (61), (62).

Secondly, in contrast to the generic arguments of Ref. [23], in the case we are studying here, there is the constraint Eq. (52), implemented in an effective action treatment through the Lagrange multiplier term, Eq. (53). There are two distinct phases in the problem, as a result of the quantum treatment of the constraint; we discuss them below.

(a) The phase in which the constraint is irrelevant, i.e. $\langle\lambda(x)\rangle = 0$, in which case the vector Dirac-Born-Infeld field is massless. This is the case for an era in which sufficiently dense populations of D-particles cross our brane universe and they condense due to quantum effects, which are dominant over the stochastic population effects, Eq. (54). In this case, the quantum effects on the recoil vector field condensate dominate and one may write, according to our previous discussion and taking into account the isometry structure Eq. (62), that

$$\langle F_{\mu\alpha}\, F_{\nu\beta} g^{\alpha\beta}\rangle_{\rm Vac} = \frac{\tilde{\alpha}(t)}{4}\, g_{\mu\nu}\ , \tag{67}$$

where the quantum condensate $\tilde{\alpha}(t)$ can be even negative as we have seen previously (*cf.* Eq. (66)). In this phase, the recoil vector field represents quantum fluctuations of the D-particles in the absence of any matter string excitations. The nomenclature "recoil" may therefore be misleading but we keep it for uniformity purposes.

(b) The phase in which $\langle\lambda(x)\rangle \equiv \mathcal{M}^2/2 > 0$, in which case the vector field becomes massive. This is the phase where contributions to the growth of galaxies will come from, and is assumed to characterise the era of structure formation. Quantum effects on possible condensates are assumed sub-dominant. One has the *classical* effects on the condensates Eqs. (49), (54), (55) and (56) that dominate in this case.

1. *de Sitter phase and quantum vacuum condensates*

We concentrate on the phase **(a)**, which as we shall demonstrate, is compatible with a late-era de Sitter phase for our Universe, even after including the coupling of the target space-time curvature to the Dirac-Born-Infeld terms¡ Eq. (39), which was ignored in the analysis of Ref. [23]. To this end, we concentrate on the case where the condensate $\tilde{\alpha}$ is *constant* in time, *small* in magnitude, while the condensate $\tilde{\beta} = 0$, so that an expansion up to order $\tilde{\alpha}$ in the effective action is sufficient. To this order our perturbative equations (58), (59) and (60) suffice. Consider the dilaton equation (60) first. Upon assuming the formation of quantum condensates, Eq. (67), we observe that this equation implies that the scalar curvature of the space-time reads

$$R = \frac{1}{\alpha\left(\frac{2\,T_3}{g_{s0}}\, e^{3\phi_0} - \tilde{\alpha}\right)}\left[\frac{6}{g_{s0}}\, T_3\, e^{5\phi_0} + \frac{4\tilde{\Lambda}}{\kappa_0}\, e^{4\phi_0}\right]\ . \tag{68}$$

This can be a *positive constant*, as appropriate for a de Sitter space-time, provided

$$\frac{2\,T_3}{g_{s0}}\, e^{3\phi_0} - \tilde{\alpha}\, >\, 0\ . \tag{69}$$

If we assume a de Sitter maximally symmetric form for the Riemann curvature tensor, corresponding to a Hubble constant, $H_{\rm I} = {\rm const} > 0$, namely,

$$R_{\mu\nu\rho\sigma} = H_{\rm I}^2\, (g_{\mu\rho}\, g_{\nu\sigma} - g_{\mu\sigma}\, g_{\nu\rho})\ ,$$

we obtain from Eq. (68) the following relation:

$$H_{\rm I}^2 = \frac{R}{12} = \frac{1}{\alpha\left(\frac{2\,T_3}{g_{s0}}\, e^{3\phi_0} - \tilde{\alpha}\right)}\left[\frac{T_3}{2g_{s0}}\, e^{5\phi_0} + \frac{\tilde{\Lambda}}{3\kappa_0}\, e^{4\phi_0}\right] > 0\ . \tag{70}$$

From the graviton equation (59), ignoring the matter contributions $T^m_{\mu\nu}$, using Eq. (67), and considering the phase $\langle\lambda\rangle = 0$ we obtain

$$H_{\rm I}^2 = \frac{1}{6}\left[\frac{\frac{T_3}{g_{s0}}\, e^{3\phi_0} + \frac{\tilde{\Lambda}}{\kappa_0}\, e^{2\phi_0}}{\frac{1}{\kappa_0} + \alpha\left(\frac{T_3}{g_{s0}}\, e^{\phi_0} - \frac{\tilde{\alpha}}{4}\, e^{-2\phi_0}\right)}\right]\ . \tag{71}$$

From Eqs. (70) and (71) we finally obtain the value of the condensate that guarantees a de Sitter geometry in this phase, namely

$$\tilde{\alpha} = \frac{4\, e^{2\phi_0}}{\alpha\, \kappa_0}\, \left[\frac{1 + \alpha\, \kappa_0\, \frac{T_3}{g_{s0}}\, e^{\phi_0}\, \left(1 - \frac{1}{3}\mathcal{G}\right)}{1 - \frac{2}{3}\,\mathcal{G}}\right] \;, \qquad \mathcal{G} \equiv \frac{\frac{T_3}{g_{s0}}\, e^{\phi_0} + \frac{\tilde{\Lambda}}{\kappa_0}}{\frac{T_3}{2\, g_{s0}}\, e^{\phi_0} + \frac{\tilde{\Lambda}}{3\, \kappa_0}} \;. \tag{72}$$

Thus, the condensate is proportional to the factor

$$\frac{e^{2\phi_0}}{\alpha\, \kappa_0} = \frac{6 V^{(6)}}{\pi^2 (g_{s0} e^{-\phi_0})^2} \;, \tag{73}$$

using Eq. (26) and adopting the value of α that appears in the example of Ref. [16]. This factor depends on the size of the compactification volume.

For mathematical consistency of our solution, the condensate $\tilde{\alpha}$ and the curvature, *i.e.* the Hubble constant $H_{\rm I}$ (70) have to be sufficiently small, which is easily guaranteed from Eq. (72) for sufficiently large negative values of ϕ_0. Such large negative values of the dilaton may characterise late epochs of the universe. For instance, in the linear (run-away) dilaton scenario, in terms of the FLRW time in the Einstein frame, the dilaton assumes large negative values, $\phi \sim -\ln t \gg -1$ for large times $t \to \infty$, and its variation is suppressed by $\dot{\phi} \sim -1/t \to 0$, so it may be considered approximately constant. This could be a phase where the above conditions for the formation of small condensates are satisfied, in which case the universe will enter a late de Sitter phase.

From our microscopic D-particle foam point of view, such a late phase may occur when matter in the brane is sufficiently dilute due to the brane world cosmic expansion, and the brane universe passes again a bulk area with sufficiently dense D-particle populations. The classical recoil effects of the D-particles, due to their interaction with matter strings on the brane, become then sub-dominant to their *quantum vacuum fluctuation* effects. In fact, as already mentioned previously, in our microscopic models of D-foam Refs. [6, 7], in such densely populated bulk areas, the bulk cosmological constant $\tilde{\Lambda}$ may be negative,

$$\tilde{\Lambda} = -|\tilde{\Lambda}| \, < \, 0 \quad \text{for dense populations of nearby bulk D} - \text{particles} \;, \tag{74}$$

since the negative contributions to the brane vacuum energy from the nearby bulk D-particles dominate. It is not inconceivable then, although admittedly this requires some fine tuning, that the bulk density populations are such that in order of magnitude

$$\mathcal{G} \gg -1 \qquad , \quad \text{e.g} \;\; \frac{T_3}{g_{s0}}\, e^{\phi}_0 \sim \frac{2\,|\tilde{\Lambda}|}{3\, \kappa_0} \;, \tag{75}$$

in which case Eq. (72) implies:

$$\tilde{\alpha} \simeq 2\, e^{2\phi_0}\, \frac{T_3}{g_{s0}}\, e^{\phi_0} \sim e^{2\phi_0}\, \frac{4\,|\tilde{\Lambda}|}{3\, \kappa_0} \;, \tag{76}$$

and the condensate $\tilde{\alpha} \ll 1$ (as required for consistency of the approach) for $|\tilde{\Lambda}|/\kappa_0 \ll 1$ and *finite* values of ϕ_0.

In such a case, the formation of the condensate may be understood as *stabilising* the brane vacuum. Indeed, as follows from Eq. (59), upon the formation of constant *quantum vacuum* condensates $\tilde{\alpha}$, the dark energy terms in the brane effective action assume the form

$$\text{Brane Dark Energy} \sim \tilde{\alpha} + e^{2\phi_0}\left(\frac{T_3}{g_{s0}}\, e^{\phi_0} - \frac{|\tilde{\Lambda}|}{\kappa_0}\right) \;. \tag{77}$$

In the absence of a condensate, the brane vacuum energy would be *negative*, of order $-|\tilde{\Lambda}|/\kappa_0$ and his would indicate an instability of the vacuum. The true stable vacuum of the theory would then be the one in which the condensate forms, with the value Eq. (76), which implies a *positive* (de Sitter-type) *vacuum energy*, Eq. (77), of order $e^{2\phi_0}\, \frac{|\tilde{\Lambda}|}{\kappa_0} \, > \, 0$, for any value of ϕ_0. This would guarantee the validity of our arguments on a D-vacuum-induced de Sitter phase even in the case of phenomenologically relevant values of the string coupling, reproducing the standard model couplings today. However, in the context of our microscopic model derived from strings, this case would correspond (*cf.* Eq. (68)) to a large Hubble parameter (and hence curvature) of order $M_s^2 = 1/\alpha'$, so our lowest-order approximations would not be valid (this of course does not preclude our discussion on the stabilising properties of the condensate from being correct in a non-perturbative string context). Hence, it seems that, at least within our string effective context, one cannot avoid making the assumption of large negative values of ϕ_0 in the discussion of late-era D-foam vacuum fluctuations condensates.

2. Matter-dominated phase and statistical contributions of D-particle populations to vector-field condensates

We next proceed to discuss the massive phase of the recoil vector field, where matter effects dominate, and in this sense the contributions of the classical effects of recoil velocity fluctuations, obtained after averaging over D-particle populations, to condensates of the vector field strengths dominate over quantum fluctuation effects of D-particles.

Assuming only electric-field type backgrounds, which is the background considered so far, we do have for such classical averages:

$$\ll F_{0\,\nu}\, F_0{}^{\nu} \gg = \ll \sum_{i=1}^{3} E_i^2 \gg \equiv 3\tilde{\sigma}^2(t) > 0 \ ,$$
$$\ll F_{i\,0}\, F^0_j \gg = - \ll E_i\, E_j \gg \equiv -g_{ij}\, \tilde{\sigma}^2(t) \ , \tag{78}$$

where we assumed isotropic recoil fluctuations for simplicity, and we denoted generically the recoil fluctuations of the field strength (averaged over D-particle populations) as $\tilde{\sigma}^2(t)$, which is a function of the cosmic time. We note that scale factor dependent contributions are included, as in Eqs. (49), (56). The metric g_{ij} denotes the FLRW space-time background. Using Eqs. (49), (50), we get

$$\tilde{\sigma}^2(t) = 4H^2\, a^2(t)\sigma_0^2 = 4\frac{H^2}{a(t)}\,\beta \quad , \quad H = \frac{\dot{a}}{a} \ . \tag{79}$$

We next notice that, irrespectively of the specific form of the condensate, in case these classical (statistical) effects are dominant over the true quantum ones, Eqs. (62),(78) imply

$$\alpha_{\rm t}(t) = -12\tilde{\sigma}^2(t) \, < \, 0 \ , \quad \alpha_{\rm s}(t) = +\frac{1}{3}\alpha_{\rm t}(t) = -4\tilde{\sigma}^2(t) \tag{80}$$

and the corresponding Dirac-Born-Infeld analysis Eq. (65) yields *positive energy density*

$$\rho^{\rm DBI-recoil} = \frac{3\,\lambda}{2}\,\tilde{\sigma}^2(t) \, > 0 \ , \tag{81}$$

but *negative pressure*

$$p^{\rm DBI-recoil} = -\frac{\lambda}{2}\,\tilde{\sigma}^2(t) = -\frac{1}{3}\,\rho^{\rm DBI-recoil} \ , \tag{82}$$

and an equation of state

$$w_{\rm DBI-recoil} \simeq -\frac{1}{3} \ . \tag{83}$$

This is the limiting case for inducing acceleration in the universe, which is fine given that we are in the matter-dominated phase, where the decelerating effects of matter cannot be ignored. Hence, all this result tells us is that the matter-induced recoil effects of D-particles are not sufficient to create a cosmological-constant-type accelerating universe, unlike the quantum fluctuation effects [5].

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[5] However in our model there are many other factors that contribute to the equation of state and to acceleration, bulk D-particles plus closed string sector quantum effects contributing to brane tension. Moreover, in our analysis in this work, we have ignored the presence of recoil-induced "magnetic-field" contributions. We do notice at this stage that for stability reasons on a D-brane universe, magnetic-field type backgrounds are also necessary. In our recoil case this is easily achieved, as discussed in Ref. [24], by considering angular momentum type contributions in the world-sheet deformations (we give them here in flat space-time for brevity), which are still compatible with the logarithmic conformal algebras,

$$V^{\rm angular\ momentum} \propto \int_{\partial\Sigma} u^i \epsilon_{ijk} X^j \Theta(t-t_0)\partial_n X^k = \int_{\Sigma} \epsilon^{\alpha\beta}\left(u^i\epsilon_{ijk}\Theta(t-t_0)\partial_\alpha X^j \partial_\beta X^k + \mathcal{O}(\delta(t-t_0))\right) \ , \tag{84}$$

and yield magnetic-field type background in target space $B_i \sim u_i$, with u_i the recoil velocity of the defect. Such backgrounds do contribute upon performing statistical averages over D-particle populations, terms $\ll B_iB_j \gg \sim \ll E_iE_j \gg \sim \delta_{ij}\,\tilde{\sigma}^2(t)$, thereby making the situation entirely analogous to the one considered in Ref. [23] and described briefly above. The result of these classical contributions then in this case is a fluid for the vector field describing the dynamics of D-particle-recoil which has *positive* pressure and energy and thus, in the case the constraint Eq. (52) is ignored, behaves as a relativistic matter fluid with $p^{\rm DBI} = (1/3)\rho^{\rm DBI} > 0$. We shall not consider, though, such magnetic field contributions in what follows but we felt stating their potentially important role for completeness.

We will use Eqs. (62), (80) in the equations of motion Eqs. (60) and (59), but keeping the matter stress energy tensor terms $T^{\rm m}_{\mu\nu}$.

Let us first analyse the graviton equation (59). We have

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} + g_{\mu\nu}\, 8\pi {\rm G}_{\rm eff}\, \Lambda_{\rm eff} = 8\pi {\rm G}_{\rm eff}\, T^{m}_{\mu\nu} + 8\pi {\rm G}_{\rm eff}\, \mathcal{F}_{\mu\nu} \tag{85}$$

whereby

$$\begin{aligned}
{\rm G}_{\rm eff} &\equiv \frac{1}{8\pi}\left(\frac{1}{\kappa_0} + \alpha\, \frac{T_3}{g_{\rm s0}}\, e^{\phi_0} + \frac{\alpha e^{-2\phi_0}}{4} F^{\alpha\beta}F_{\alpha\beta}\right)^{-1} , \\
\Lambda_{\rm eff} &\equiv \frac{1}{8}\, F^{\alpha\beta}F_{\alpha\beta} + \frac{\tilde{\Lambda}\, e^{2\phi_0}}{2\kappa_0} + \frac{T_3 e^{3\phi_0}}{2 g_{\rm s0}} - \frac{\langle\lambda(x)\rangle}{2}\left[A_\alpha\, A^\alpha + \frac{|T_3|}{g_{\rm s0}} 2\pi\alpha'\, e^{-\phi_0}\right] , \\
\text{and}\ \ \mathcal{F}_{\mu\nu} &\equiv \frac{1}{2} g^{\sigma\lambda} F_{\mu\lambda} F_{\nu\sigma}(1 - \alpha e^{-2\phi_0} R) - \frac{\alpha e^{-2\phi_0}}{4}\left\{ g_{\mu\nu}\nabla^2\left[F^{\alpha\beta}F_{\alpha\beta}\right] - \nabla_\mu\nabla_\nu\left[F^{\alpha\beta}F_{\alpha\beta}\right]\right\} - \langle\lambda(x)\rangle A_\mu\, A_\nu\ , \\
\text{with}\ \ \langle\lambda(x)\rangle &= -\frac{g_{s0} e^{\phi_0}}{|T_3| 4\pi\alpha'} A^\mu \left[F_{\nu\mu}(1 - \alpha e^{-2\phi_0} R)\right]^{;\nu} .
\end{aligned} \tag{86}$$

The form of the lagrange multiplier λ is found by contracting the vector equation of motion, Eq. (58), with A^μ and then applying the constraint Eq. (52).

Next, we use the dilaton constraint Eq. (60) to rewrite the term $\tilde{\Lambda}\, e^{2\phi_0}/\kappa_0$ in $\Lambda_{\rm eff}$. For convenience we split $\Lambda_{\rm eff}$ into two parts such that

$$\begin{aligned}
\Lambda_{\rm eff} &= \Lambda^{(0)}_{\rm eff} + \Lambda^{(1)}_{\rm eff}\ , \\
\text{with}\ \ \Lambda^{(0)}_{\rm eff} &\equiv -\frac{T_3 e^{3\phi_0}}{4 g_{\rm s0}}\left(1 - \alpha e^{-2\phi_0} R\right) - \frac{\langle\lambda(x)\rangle}{2}\left[A_\alpha\, A^\alpha + \frac{|T_3|}{g_{\rm s0}} 2\pi\alpha'\, e^{-\phi_0}\right] , \\
\text{and}\ \ \Lambda^{(1)}_{\rm eff} &\equiv \frac{F^{\alpha\beta}F_{\alpha\beta}}{8}\left(1 - \alpha e^{-2\phi_0} R\right) + \frac{3\alpha e^{2\phi_0}}{8}\nabla^2\left[F^{\alpha\beta}F_{\alpha\beta}\right] .
\end{aligned} \tag{87}$$

This allows the separation of those parts of $\Lambda_{\rm eff}$ which depend on σ_0^2 terms from those which are σ_0^2-independent. Similarly, ${\rm G}_{\rm eff}$ can also be split as

$$\begin{aligned}
{\rm G}_{\rm eff} &= \left[G^{(0)}_{\rm eff} + G^{(1)}_{\rm eff}\right] , \\
\text{with}\ \ G^{(0)}_{\rm eff} &\equiv \frac{1}{8\pi}\left[\frac{1}{\kappa_0} + \frac{\alpha T_3 e^{\phi_0}}{g_{\rm s0}}\right]^{-1} , \\
\text{and}\ \ G^{(1)}_{\rm eff} &\equiv \frac{\alpha e^{-2\phi_0} F^{\alpha\beta}F_{\alpha\beta}}{32\pi}\left[\frac{1}{\kappa_0} + \frac{\alpha T_3 e^{\phi_0}}{g_{\rm s0}}\right]^{-2} .
\end{aligned} \tag{88}$$

The constant $G^{(0)}$ plays the role of the gravitational constant in this scenario. The background configuration of the fields has already been stated in Eq. (45) and Eq. (50). This then allows us to find the background solutions to the equations of motion.

B. The Background Solution

For the background, the configuration of the gauge and graviton fields has been explained in the earlier sections and was shown to be

$$\begin{aligned}
g_{00} &= -1\ , \\
g_{ij} &= a^2(t)\delta_{ij}\ , \\
A_0 &= \left[\frac{2\pi\alpha' |T_3| e^{-\phi_0}}{g_{s0}} + u^2 a^2(t)\right]^{1/2} , \\
A_i &= -a^2(t) u_i\ .
\end{aligned} \tag{89}$$

Furthermore, assuming that the matter in our system can be treated as an ideal fluid, it is found that

$$T^{\mathrm{m}\mu}{}_{\nu} = (\rho + P) w^{\mu} w_{\nu} + P\delta^{\mu}{}_{\nu} \ , \tag{90}$$

where ρ and P are the density and pressure of the fluid and w_μ is its velocity vector field, normailised to be timelike. Thus

$$\begin{aligned} T^{\mathrm{m}0}{}_{0} &= -\rho \ , \\ T^{\mathrm{m}i}{}_{j} &= P\delta^{i}{}_{j} \ . \end{aligned} \tag{91}$$

With the background configuration defined, we begin looking at the background solutions by examining the vector equation Eq. (58). By contracting Eq. (58) with A^μ is possible to find the form of the Lagrange multiplier in the background. This is useful as it can then be used in the other equations of motion when calculating their background forms. Thus from Eq. (58) we can see that

$$\lambda(x)\frac{2\pi\alpha'|T_3|e^{-\phi_0}}{g_{s0}} = -\frac{A^\mu}{2}\left[F_{\nu\mu}(1-\alpha e^{-2\phi_0}R)\right]^{;\nu} \ . \tag{92}$$

With our background and after taking the vev of the recoil velocity in accordance with Eq. (54), it is found that the above relation gives

$$\langle\lambda\rangle\frac{2\pi\alpha'|T_3|e^{-\phi_0}}{g_{s0}} = 3\sigma_0^2 a^2\left[\frac{\ddot{a}}{a} + 2H^2 - 6\alpha e^{-2\phi_0}\left(\frac{H\dddot{a}}{a} + \frac{\ddot{a}^2}{a^2} + \frac{4H^2\ddot{a}}{a}\right)\right] \ . \tag{93}$$

Next we can see that the vector equation of motion, Eq. (58), is linear in the gauge field. Thus, when looking at the spatial component of the equation, all the terms will be proportional to u_i and by Eq. (54) such terms will be zero so the spatial component of the equation vanishes. The temporal component remains non zero as A_0 from Eq. (89) is not proportial to u_i. Thus we find that the temporal component of the vector equation in the background yeilds

$$0 = 3\sigma_0^2 a^2\left[\frac{g_{s0}}{2\pi\alpha'|T_3|e^{-\phi_0}}\right]^{1/2}\left[\frac{\ddot{a}}{a} + 2H^2 - 6\alpha e^{-2\phi_0}\left(\frac{H\dddot{a}}{a} + \frac{\ddot{a}^2}{a^2} + \frac{4H^2\ddot{a}}{a}\right)\right] \ . \tag{94}$$

Next we can look at the dilaton equation, Eq.(60). This gives

$$0 = \left[12\alpha\sigma_0^2 a^2 e^{-2\phi_0}H^2 + \frac{T_3 e^{\phi_0}\alpha}{g_{s0}}\right]\left(\frac{\ddot{a}}{a} + H^2\right) + 12\alpha\sigma_0^2 a^2 e^{2\phi_0}\left[\frac{\ddot{a}^2}{a^2} + \frac{H\dddot{a}}{a} + \frac{3H^2\ddot{a}}{a}\right] \tag{95}$$

However a more convenient way to use the dilaton equation is to use it as a constraint in the graviton equation, as was done in Eq.(87). Thus when moving on to the graviton equation we shall be using the dilaton constrained version of this equation. When considering the background graviton solution, we can write the modified Einstein equation as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G_{\mathrm{eff}}^{(0)}\left[T_{\mu\nu}^{\mathrm{m}} - \Lambda_{\mathrm{eff}}^{(0)}g_{\mu\nu}\right] + T_{\mu\nu}^{\mathrm{DBI}} \ . \tag{96}$$

Here $T_{\mu\nu}^{\mathrm{DBI}}$ represents those terms coming from the effects of the Dirac-Born-Infeld field; it is defined to be

$$T_{\mu\nu}^{\mathrm{DBI}} = 8\pi G_{\mathrm{eff}}^{(0)}\left(\mathcal{F}_{\mu\nu} - \Lambda_{\mathrm{eff}}^{(1)}g_{\mu\nu}\right) - 8\pi G_{\mathrm{eff}}^{(1)}\left(T_{\mu\nu}^{\mathrm{m}} - \Lambda_{\mathrm{eff}}^{(0)}g_{\mu\nu}\right) \ . \tag{97}$$

Note that for the background solution the $\langle\lambda(x)\rangle$ term in the definition of $\Lambda_{\mathrm{eff}}^{(0)}$ is zero upon application of the constraint equation. In this case we can write the modified continuity equation as

$$\nabla^\mu\left[T_{\mu\nu}^{\mathrm{m}} - \Lambda_{\mathrm{eff}}^{(0)}g_{\mu\nu} + \frac{T_{\mu\nu}^{\mathrm{DBI}}}{8\pi G_{\mathrm{eff}}^{(0)}}\right] = 0 \ . \tag{98}$$

For the FLRW background we are considering and denoting by ρ^{tot} the total density contributions coming from matter (ρ^{m}), the DBI part (ρ^{DBI}) and any dark fluid (ρ^{Λ}), we find, upon assuming a perfect fluid form of stress energy tensor, that

$$\frac{\mathrm{d}\rho^{\mathrm{tot}}}{\mathrm{d}a} + \frac{3\rho^{\mathrm{tot}}}{a} + \frac{3P^{\mathrm{tot}}}{a} = 0 \ , \tag{99}$$

just as in the standard gravitation case. In order to investigate the effects of $\rho^{\rm DBI}$ in the matter dominated phase of structure formation we note the following. The contribution from ρ^{Λ} can be neglected in this phase. Moreover, for pressureless matter, the total pressure, $P^{\rm tot}$, has only Dirac-Born-Infeld contributions (including the constraint term Eq. (51)) such that $P^{\rm tot} = P^{\rm DBI}$. The equation of state of the total fluid can then be used leading to

$$P^{\rm tot} = -\frac{\rho^{\rm DBI}}{3} \ . \tag{100}$$

We thus get the following first order differential equation

$$\frac{{\rm d}\rho^{\rm tot}}{{\rm d}a} + \frac{3\rho^{\rm tot}}{a} = \frac{\rho^{\rm DBI}}{a} \ , \tag{101}$$

with solution

$$\rho^{\rm tot} = \frac{\mathcal{C}}{a^3} + \frac{1}{a^3}\int a^2 \rho^{\rm DBI} da \ , \tag{102}$$

where $\mathcal{C}$ is a constant to be determined from boundary conditions. The contribution $\rho^{\rm DBI}$ is

$$\begin{aligned}\rho^{\rm DBI} \ &= \ \frac{T_{00}^{\rm DBI}}{8\pi G^{(0)}} \\ &= \ 3\dot{a}^2\sigma_0^2\left[1-6\alpha e^{-2\phi_0}\left(H^2+\frac{\ddot{a}}{a}\right)\right] - \langle\lambda\rangle\frac{|T_3|2\pi\alpha'\,e^{-\phi_0}}{g_{\rm s0}} + 18\alpha e^{2\phi_0}\sigma_0^2\left(\dot{a}\dddot{a}+\ddot{a}^2+\frac{3\dot{a}^2\ddot{a}}{a}\right) \\ &\quad +36\alpha e^{-2\phi_0}\frac{\dot{a}^2\ddot{a}}{a}\sigma_0^2 + 6\alpha e^{-2\phi_0}\dot{a}^2\sigma_0^2\left[\frac{1}{\kappa_0}+\frac{\alpha T_3 e^{\phi_0}}{g_{\rm s0}}\right]^{-1}\left[\rho^m - \frac{T_3 e^{3\phi_0}}{4g_{\rm s0}}\left(1-6\alpha e^{-2\phi_0}\left(H^2+\frac{\ddot{a}}{a}\right)\right)\right] \ . \end{aligned} \tag{103}$$

For the (late) epoch of structure formation, we observe from Eq. (103) that the terms proportional to $\sigma_0^2 \sim a(t)^{-3}\beta$ (*cf.* Eq. (56)) are suppressed by at least one inverse power of the scale factor $a(t) \sim t^p$. Hence, to leading order, for a late era we can write

$$\rho^{\rm DBI} \simeq -\langle\lambda\rangle\,\frac{|T_3|2\pi\alpha'\,e^{-\phi_0}}{g_{\rm s0}} + \mathcal{O}\left(\frac{1}{a}\right) \ . \tag{104}$$

The background form of the Lagrange multiplier was calculated previously in Eq.(93). Upon examination again we see that the Lagrange multiplier terms are suppressed by at least one inverse power of the scale factor. Thus for late time eras, the DBI density contribution can be taken to be negligible.

The total energy density $\rho^{\rm total} = \rho^{\rm m} + \rho^{\rm DBI}$ is determined from Eq. (102). Matter dominance already required that the DBI terms should be suppressed compared to the standard $1/a^3$ terms, which we have now explicitly shown is the case. Thus, to a good approximation the matter dominated era scale factor, scaling with the FLRW time, can be taken to be $a_{\rm matter}(t) \sim t^{2/3}$, as in the standard gravitational case. Lastly then, we can set the constant of integration, $\mathcal{C}$, to be equal to the ordinary matter density today, $\rho_{\rm m0}$. We find it convenient to work in units of the critical density today $\rho_{\rm cr} = 1$, in which $\rho_{\rm m0} = \Omega_{\rm m}$ in the standard notation. Thus, to leading order for the era of structure formation where the effect of the cosmological constant can be neglected,

$$\rho^{\rm tot} = \frac{\Omega_{\rm m}}{a^3} + \mathcal{O}\left(\frac{1}{a}\right) \ . \tag{105}$$

Some important comments are now in order concerning the nature of the Ω_m. In our model, as already mentioned, there are contributions to Ω_m from the D-particles (due to their masses) as well as the conventional dark matter components (if there are any). However, due to the competing effects of the nearby and far-away bulk D-particles (*cf.* Eqs. (2), (3)), there may be significant cancellations in the D-particle contribution to Ω_m. As we shall discuss in Section VI, to isolate the effects of D-matter we shall consider the extreme case where Ω_m is only baryonic and the D-particles play an important role in growth mainly due to their interaction with neutral matter, which provides the seed for the appearance of the recoil vector field and its associated perturbations.

Finally we can write down the explicit form of the density and pressure equations coming from the dilaton constrained graviton equation of motion. Here we shall not assume any particular era of the universe and give the full general equations, unlike previously when considering the continuity equation where only the matter domination era

was considered and the effect of ρ_Λ could be neglected. Below we have also incorporated the form of the Lagrange multiplier, Eq. (93).

$$
\begin{aligned}
\rho &= 3H^2\left[\frac{1}{\kappa_0}+\frac{\alpha T_3 e^{\phi_0}}{g_{s0}}\right]+\frac{T_3 e^{3\phi_0}}{4g_{s0}}\left[1-6\alpha e^{-2\phi_0}\left(H^2+\frac{\ddot{a}}{a}\right)\right]\\
&\quad+\sigma_0^2 a^2\left[3H^2+\frac{3\ddot{a}}{a}-18\alpha e^{-2\phi_0}\left(\frac{\ddot{a}^2}{a^2}+\frac{H\ddot{a}}{a}+\frac{5H^2\ddot{a}}{a}\right)-18\alpha e^{2\phi_0}\left(\frac{\ddot{a}^2}{a^2}+\frac{H\ddot{a}}{a}+\frac{3H^2\ddot{a}}{a}\right)\right]\\
-P &= \left(H^2+\frac{2\ddot{a}}{a}\right)\left[\frac{1}{\kappa_0}+\frac{\alpha T_3 e^{\phi_0}}{g_{s0}}\right]+\frac{T_3 e^{3\phi_0}}{4g_{s0}}\left[1-6\alpha e^{-2\phi_0}\left(H^2+\frac{\ddot{a}}{a}\right)\right]\\
&\quad+\sigma_0^2 a^2\left[H^2-6\alpha e^{-2\phi_0}\left(\frac{2\ddot{a}^2}{a^2}+\frac{2H\ddot{a}}{a}+\frac{9H^2\ddot{a}}{a}+H^4\right)-18\alpha e^{2\phi_0}\left(\frac{\ddot{a}^2}{a^2}+\frac{H\ddot{a}}{a}+\frac{3H^2\ddot{a}}{a}\right)\right] \qquad (106)
\end{aligned}
$$

Here we can see again that the contribution of those terms proportional to σ_0^2 are supressed in late eras by at least one power of the scale factor. We do notice, however, that these contributions, to leading order in powers of the scale factor, *i.e.* the terms of the form $\sigma_0^2 a^2 H^2$ in (106), do satisfy the equation of state of the DBI fluid (100), if those terms were dominant. The constant terms $\frac{T_3 e^{3\phi_0}}{4g_{s0}}$, on the other hand, play the role of the cosmological constant. Thus we reproduce, as a further consistency check of our perturbative approach, the standard contributions to the density and the pressure in the matter era, obtained previously, with our extra factors coming from the DBI part of the action all being suppressed by the scale factor. This allows us to treat their effects as subleading in the late eras of the Universe evolution, to which we shall restrict ourselves below.

C. The Perturbed Solution

Let us now consider the perturbed metric; it reads

$$g_{00} = -1 + 2\Psi \ , \quad g_{ij} = a^2(t)\,(1+2\Phi)\,\delta_{ij} \ . \qquad (107)$$

The perturbed vector field assumes the form

$$A_\mu = a(t)\left(-a(t)u_\mu + \tilde{A}_\mu\right) \ , \qquad (108)$$

where we used the background solution Eq. (50).

Caution should be exercised when we parametrise the vector field perturbations, which should respect the time-like constraint for the vector field Eq. (52). One should use the gauge covariance associated with the general coordinate diffeomorphisms in order to parametrise the temporal components of the vector perturbations in terms of those of the metric [25]. Below we summarise briefly the situation and state the final result, relevant to our purposes here. For details we refer the reader to Ref. [25]. The reader should recall that, in contrast to the TeVeS models, in our case the dilaton is constant and its perturbations are ignored as sub-leading for reasons stated previously. Moreover, the spatial components of the background vector field are non vanishing, except when considering averages of Eq. (55) over populations of D-particles in the foam. Nevertheless, these do not affect the general form of the vector perturbations, which are of the same form as for TeVeS models.

The perturbation $\tilde{A}_\mu$ of the time-like vector field A_μ, Eq. (108), in a conformal metric background such as

$$g_{00} = a^2(\eta) \quad , \quad g_{ij} = a^2(\eta) \qquad (109)$$

(where conformal time η is related to cosmological time t in the standard way) can be decomposed as follows:

$$\tilde{A}_\mu = -\Xi t_\mu + \overline{q}^\nu{}_\mu \overline{\nabla}_\nu \zeta + \beta_\mu \ , \qquad (110)$$

where over-line denotes quantities pertinent to the conformal background, t_μ is a time-like unit Killing vector tangent to the geodesics, $t^\mu t_\mu = -1$, such that $t^\mu \beta_\mu = 0$, and $\overline{q}^\mu{}_\nu = \delta^\mu{}_\nu + t^\mu\, t_\nu$ is a projector appropriate for the conformal metric [25], such that $\overline{q}^\mu{}_\nu A^\nu = 0$, $\overline{q}^\mu{}_\alpha \overline{q}^\alpha{}_\nu = \overline{q}^\mu{}_\nu$. The fields ζ and Ξ are scalar modes, and β_μ are vector modes such that $\overline{q}^{\mu\nu}\overline{\nabla}_\mu \beta_\nu = 0$. There are two independent vector modes, as a result of the constraint Eq. (52) and gauge invariance under diffeomorphisms. The scalar mode ζ satisfies [25]

$$\overline{\nabla}^2 \zeta = \vec{\overline{\nabla}} \cdot \vec{\tilde{A}} \ . \qquad (111)$$

There is gauge freedom in transforming the various fields under general coordinate transformations, which results in simplifications (upon gauge fixing) of the vector perturbations. To this end, let one consider an infinitesimal diffeomorphism $\xi_\mu = \frac{1}{a}\widehat{\xi}_\mu$, where a is the scale factor of the conformal metric. The vector $\widehat{\xi}_\mu$ can be decomposed as $\widehat{\xi}_\mu = -\xi t_\mu + \overline{q}^\nu{}_\mu \overline{\nabla}_\nu \psi + \omega_\mu$, with $t^\mu \omega_\mu = 0$, which contains two scalar modes $\xi = t^\mu \widehat{\xi}_\mu$ and ψ, such that $\overline{\nabla}^2 \psi = \overline{q}^{\mu\nu}\overline{\nabla}_\mu \widehat{\xi}_\nu$, and two vector modes ω_μ with the property $\overline{q}^{\mu\nu}\overline{\nabla}_\mu \omega_\nu = 0$. Under such diffeomorphisms, the vector perturbation $\tilde{A}_\mu$ transforms as

$$\tilde{A}'_\mu = \tilde{A}_\mu + \frac{1}{a}\nabla_\mu \xi \ ; \tag{112}$$

we recall that we ignore dilaton perturbations, which would in general contribute.

From Eqs. (110) and (112) we observe that the scalar mode ζ transforms as

$$\zeta' = \zeta + \frac{1}{a}\xi \ , \tag{113}$$

the scalar Ξ transforms as

$$\Xi' = \Xi + \frac{1}{a} t^\mu \overline{\nabla}_\mu \xi \ . \tag{114}$$

while the vector modes β_μ remain gauge invariant.

Using the gauge freedom endowed in Eqs. (113), (114) we may then gauge fix the perturbations of the vector field, by making the gauge choice [25]

$$\Xi = -\Psi \ , \quad \beta_\mu = 0 \ , \tag{115}$$

where Ψ is defined in the metric perturbation Eq. (107). Thus, the form of the (gauge fixed) vector perturbation reads

$$\tilde{A}_\mu = \left[\frac{|T_3|e^{-\phi_0}}{g_{s0}} 2\pi\alpha'\right]^{1/2} \left(\Psi, \vec{\tilde{A}}\right), \tag{116}$$

with $\vec{\tilde{A}}$ satisfying Eq. (111), or equivalently

$$\vec{\overline{\nabla}}\zeta = \vec{\tilde{A}} \ , \tag{117}$$

setting constants to zero. A shift back to the Robertson-Walker metric using a standard co-ordinate transformation leads to

$$\tilde{A}_\mu = \left[\frac{|T_3|e^{-\phi_0}}{g_{s0}} 2\pi\alpha'\right]^{1/2} \left(\frac{\Psi}{a(t)}, \vec{\tilde{A}}\right). \tag{118}$$

Here the $\left[\frac{|T_3|e^{-\phi_0}}{g_{s0}} 2\pi\alpha'\right]^{1/2}$ factor is included from dimensional considerations. The last perturbations which need to be defined are the perturbations of the matter energy momentum tensor. In our system Eq.(90) can be perturbed to find that

$$\delta T^{\mathrm{m}0}{}_0 = -\delta\rho \ , \quad \delta T^{\mathrm{m}i}{}_j = \delta P \delta^i{}_j \ . \tag{119}$$

We can now begin to examine the perturbed equations of motion, beginning with the graviton equation. In doing so, it is useful to move into Fourier space, such that we get the following conversion

$$\partial_i = ik_i \ . \tag{120}$$

This converts the partial differential equations into ordinary ones, making them easier to manipulate. The 00 com-

ponent of the graviton equation yields the following

$$
\begin{aligned}
T^{\mathrm{m}0}{}_{0} \;=\; & -\left[\frac{1}{\kappa_0}+\frac{\alpha T_3 e^{\phi_0}}{g_{s0}}\right]\left[\frac{2k^2}{a}\Phi+6H^2\Psi+6H\dot{\Phi}\right] \\
& +6\sigma_0^2 a^2\Phi\left\{\frac{\ddot{a}}{a}+H^2-6\alpha e^{-2\phi_0}\left[\frac{\ddot{a}^2}{a^2}+\frac{H\dddot{a}}{a}+\frac{4H^2\ddot{a}}{a}-H^4+\frac{k^2}{3a^2}\left(2H^2-\frac{\ddot{a}}{a}\right)\right]\right\} \\
& +\Psi\left\{\frac{|T_3|(2\pi\alpha')e^{-\phi_0}}{g_{s0}}\frac{k^2}{2a^2}\left[1-6\alpha e^{-2\phi_0}\left(H^2+\frac{\ddot{a}}{a}\right)\right]-6\sigma_0^2a^2\left[\frac{2\ddot{a}}{a}+3H^2-\frac{k^2(1+2/a^2)}{8a^2}\right.\right. \\
& \qquad \left.\left. -\,6\alpha e^{-2\phi_0}\left(\frac{3\ddot{a}^2}{a^2}+\frac{3H\dddot{a}}{a}+\frac{12H^2\ddot{a}}{a}-2H^4-\frac{k^2}{4}\left[\frac{\ddot{a}}{a}\left(\frac{1}{a^2}+\frac{7}{6}\right)+H^2\left(\frac{1}{a^2}-\frac{13}{6}\right)\right]\right)\right]\right\} \\
& -3\sigma_0^2a^2\dot{\Phi}\left\{H-6\alpha e^{-2\phi_0}\left(\frac{11H\ddot{a}}{a}-3H^3+\frac{2Hk^2}{3a^2}\right)\right\}-3\sigma_0^2a^2\dot{\Psi}\left\{H-6\alpha e^{-2\phi_0}\left(\frac{5H\ddot{a}}{a}+4H^3-\frac{Hk^2}{3a^2}\right)\right\} \\
& +18\sigma_0^2a^2\alpha e^{-2\phi_0}\ddot{\Phi}\left\{\frac{\ddot{a}}{a}+4H^2\right\}+18\sigma_0^2a^2\alpha e^{-2\phi_0}\ddot{\Psi}+18\sigma_0^2 a\dot{a}\alpha e^{-2\phi_0}\dddot{\Phi} \\
& -\frac{k^2}{4a}\left(H\zeta+\dot{\zeta}\right)\left[1-6\alpha e^{-2\phi_0}\left(H^2+\frac{\ddot{a}}{a}\right)\right]\left[\frac{2|T_3|(2\pi\alpha')e^{-\phi_0}}{g_{s0}}+3\sigma_0^2(2+a^2)\right]\;,
\end{aligned}
\tag{121}
$$

where $k^2 = k_i k^i$ is the amplitude of the spatial components of the momentum scale. Similarly, the ij component of the graviton equation yields

$$
\begin{aligned}
T^{\mathrm{m}i}{}_{j} \;=\; & -\left[\frac{1}{\kappa_0}+\frac{\alpha T_3 e^{\phi_0}}{g_{s0}}\right]\left\{\frac{k^ik_j}{a^2}(\Psi-\Phi)+\delta^i{}_j\left[\frac{k^2}{a^2}\Phi-\left(\frac{k^2}{a^2}-2H^2-\frac{4\ddot{a}}{a}\right)\Psi+2H\left(3\dot{\Phi}+\dot{\Psi}\right)+2\ddot{\Phi}\right]\right\} \\
& +2\sigma_0^2a^2\Phi\delta^i{}_j\left\{H^2-6\alpha e^{-2\phi_0}\left[\frac{2\ddot{a}^2}{a^2}+\frac{2H\dddot{a}}{a}+\frac{6H^2\ddot{a}}{a}+H^4+\frac{7H^2k^2}{6a^2}\right]\right\} \\
& +2\sigma_0^2a^2\Psi\delta^i{}_j\left\{\frac{3\ddot{a}}{a}+5H^2-\frac{k^2}{4a^4}+6\alpha e^{-2\phi_0}\left(\frac{\ddot{a}^2}{a^2}+\frac{H\dddot{a}}{a}-2H^4+\frac{k^2}{12a^4}\left[\frac{3\ddot{a}}{a}+H^2\left(3+10a^2\right)\right]\right)\right\} \\
& -12\sigma_0^2a^2\alpha e^{-2\phi_0}\dot{\Phi}\delta^i{}_j\left\{4H^3+\frac{H\ddot{a}}{a}\right\}+12\sigma_0^2a^2\alpha e^{-2\phi_0}\dot{\Psi}\delta^i{}_j\left\{3H^3+\frac{5H\ddot{a}}{a}\right\} \\
& -12\sigma_0^2\dot{a}^2\alpha e^{-2\phi_0}\left(\ddot{\Phi}-\ddot{\Psi}\right)\delta^i{}_j+6\sigma_0^2H^2\alpha e^{-2\phi_0}k^ik_j(\Phi-\Psi) \\
& +\frac{k^2\delta^i{}_j}{2a}\left(H\zeta+\dot{\zeta}\right)\left[1-6\alpha e^{-2\phi_0}\left(H^2+\frac{\ddot{a}}{a}\right)\right]\;.
\end{aligned}
\tag{122}
$$

In the above we have separated out those components which come from the standard Einstein Hilbert part of the action from those terms which appear due to the effect of the D-particles. The standard terms are those which are proportional to $1/\kappa_0 + \alpha T_3 e^{\phi_0}/g_{s0}$. At this point a very useful simplification can be made. The $T^{\mathrm{m}i}{}_j$ equation has two types of terms, those proportional to $\delta^i{}_j$ and those proportional to k^ik_j. By contracting the equation with the following tensor operator

$$\hat{k}_i\hat{k}^j-\frac{1}{3}\delta^j{}_i \tag{123}$$

all the terms proportional to $\delta^i{}_j$, including those on the left hand side of the equation are set to 0. Thus, after this operation on Eq. (122) we find that

$$\Psi-\Phi=0\;. \tag{124}$$

Thus, in our system, just as in the standard gravitational case, $\Phi = \Psi$ is a valid solution which we shall now be using throughout. As we shall show later on, the perturbations of the recoil vector field A_μ do exhibit growing modes and thus can participate in late-era structure formation, unlike the case of TeVeS-like models in [11], where there is a non-trivial difference $\Psi - \Phi \neq 0$ that is considered as the main source of the growing mode [26].

Upon using Eqs. (124), (119) and (105), we obtain for the dimensionless over-density parameter $\delta\rho/\rho$ the result:

$$\frac{\delta\rho}{\rho} = \frac{a^3}{\Omega_m}\Bigg(\left[\frac{1}{\kappa_0} + \frac{\alpha T_3 e^{\phi_0}}{g_{s0}}\right]\left[2\Phi\left(\frac{k^2}{a^2}+3H^2\right)+6H\dot{\Phi}\right]$$
$$-\Phi\Bigg\{\frac{|T_3|(2\pi\alpha')e^{-\phi_0}}{g_{s0}}\frac{k^2}{2a^2}\left[1-6\alpha e^{-2\phi_0}\left(H^2+\frac{\ddot{a}}{a}\right)\right] - 6\sigma_0^2 a^2\Bigg[\frac{\ddot{a}}{a}+2H-\frac{k^2(1+2/a^2)}{8a^2}$$
$$-6\alpha e^{-2\phi_0}\left(\frac{2\ddot{a}^2}{a^2}+\frac{2H\dddot{a}}{a}+\frac{8H^2\ddot{a}}{a}-H^4-\frac{k^2}{12a^2}\left[H^2\left(\frac{3}{a^2}+\frac{3}{2}\right)+\frac{\ddot{a}}{a}\left(\frac{3}{a^2}-\frac{1}{2}\right)\right]\right)\Bigg]\Bigg\}$$
$$+6\sigma_0^2 a^2\dot{\Phi}\left\{H-6\alpha e^{-\phi_0}\left(\frac{8H\ddot{a}}{a}-\frac{H^3}{2}+\frac{Hk^2}{6a^2}\right)\right\}-18\sigma_0^2a^2\alpha e^{-2\phi_0}\ddot{\Phi}\left\{\frac{\ddot{a}}{a}+5H^2\right\}-18\sigma_0^2 a\dot{a}\alpha e^{-2\phi_0}\dddot{\Phi}$$
$$+\frac{k^2}{4a}\left(H\zeta+\dot{\zeta}\right)\left[1-6\alpha e^{-2\phi_0}\left(H^2+\frac{\ddot{a}}{a}\right)\right]\left[\frac{2|T_3|(2\pi\alpha')e^{-\phi_0}}{g_{s0}}+3\sigma_0^2(2+a^2)\right]\Bigg)\,. \tag{125}$$

From this equation we can calculate the growth of density perturbations, provided that the evolution of the metric and vector perturbations are known. An important point to note is that the vector perturbation terms, ζ and $\dot{\zeta}$ only appear due to the $\langle\lambda(x)\rangle A_\mu A_\nu$ term of $\mathcal{F}_{\mu\nu}$ in Eq. (87). Thus the constraint term is vital in coupling the vector perturbation to the density perturbations.

VI. GROWING MODES AND LARGE-STRUCTURE FORMATION DUE TO D-PARTICLES

We now proceed to calculate the evolution of the vector field perturbations. We notice that the remaining equations of motion provide the constraints which allow the evolution of these perturbations to be calculated. Under the perturbations of the metric it is noticed that, since all the terms in the vector equation are linear in the gauge field, any perturbations which appear from the metric will always come with a u_i term. Such terms, when averaged over, are zero (as discussed previously). Thus, in our case metric perturbations only appear in the vector equation through the presence of $\Psi=\Phi$ in $\tilde{A}_0$ (*cf.* Eq. (118)). The perturbed vector equation reads

$$\ddot{\zeta}+b_1\dot{\zeta}+b_2\zeta = S[\Phi, C]\;, \tag{126}$$

where

$$\begin{aligned} S[\Phi,C] &= \frac{\dot{\Phi}}{a}+\frac{\Phi}{a}\left(H+\frac{6\alpha e^{-2\phi_0}\left[2H^3-\frac{\ddot{a}H}{a}-\frac{\dddot{a}}{a}\right]}{1-6\alpha e^{-2\phi_0}\left(H^2+\frac{\ddot{a}}{a}\right)}\right)\;, \\ b_1 &= 3H+\frac{6\alpha e^{-2\phi_0}\left[2H^3-\frac{\ddot{a}H}{a}-\frac{\dddot{a}}{a}\right]}{1-6\alpha e^{-2\phi_0}\left(H^2+\frac{\ddot{a}}{a}\right)}\;, \\ b_2 &= \frac{\ddot{a}}{a}+H^2+\frac{6\alpha e^{-2\phi_0}H\left[2H^3-\frac{\ddot{a}H}{a}-\frac{\dddot{a}}{a}\right]-2\langle\lambda(x)\rangle}{1-6\alpha e^{-2\phi_0}\left(H^2+\frac{\ddot{a}}{a}\right)}\;. \end{aligned} \tag{127}$$

where we remind the reader that $\langle\lambda(x)\rangle$ is the background Lagrange multiplier, the form of which is given in Eq. (93). In Eq. (126) the signs of terms b_1 and b_2 play a crucial role in determining if the vector modes of the perturbation enter a growing mode or a decaying one, a result which is analogous to that found in Ref. [11]. In general, growth will be seen when the contribution from b_2 is negative.

To this end, we first note that for large epochs of the Universe, namely the radiation and matter dominated eras, it can be readily seen that $\langle\lambda(x)\rangle > 0$. In our analysis below we shall assume values of $\phi_0 = \mathcal{O}(1)$, which may be phenomenologically desirable, since the string coupling $g_s = e^{\phi_0}$ determines the gauge couplings of the low-energy theory. Looking at b_2 it can be seen that in this case the $\langle\lambda(x)\rangle$ term in crucial in determining if b_2 will change sign [6].

[6] We note for completeness that in models with large and negative values of ϕ_0, the terms $\left[2H^3-\frac{\ddot{a}H}{a}-\frac{\dddot{a}}{a}\right]$ in b_2, although sub-leading at late eras of the Universe, since they contain higher orders of the scale factor and its derivatives, nevertheless could have non-negligible contributions to b_2 due to the denominator, which may be sufficiently small for sufficiently large and negative ϕ_0, such that the over all contribution of this term to b_2 is significant. In our case with $\phi_0=\mathcal{O}(1)$, though, such terms will not play any rôle in our analysis.

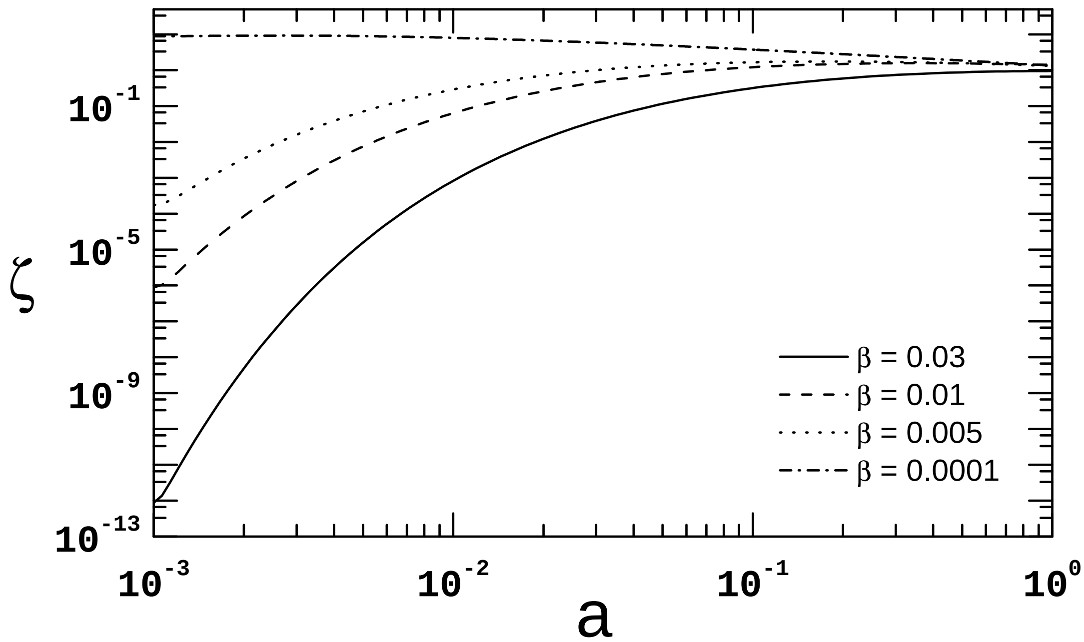

FIG. 2: Vector perturbation ζ as a function of the scale factor $a(t)$ in the matter-dominated era, for different values of the D-particle recoil velocity variance $\sigma_0^2 \propto \beta/a^3$. The string mass M_s is assumed to be 10 TeV, $\phi_0 = 1$ and $k = 0.5$ Mpc^{-1}.

Since $\langle\lambda(x)\rangle$ is proportional to the variance of the recoil velocity (*cf.* (93)), σ_0^2, we see that it is the magnitude of σ_0^2, which will determine if the vector perturbations will enter a growing mode.

Finally we examine the perturbed dilaton equation. We get

$$
\begin{aligned}
0 \;=\; & 2\Phi\left\{12\sigma_0^2 a^2\alpha\left[e^{-2\phi_0}\left(\frac{k^2H^2}{a^2}+6H^4+\frac{6H^2\ddot{a}}{a}\right)+6e^{2\phi_0}\left(\frac{H\dddot{a}}{a}+\frac{\ddot{a}^2}{a^2}+\frac{3H^2\ddot{a}}{a}\right)\right]+\frac{\alpha T_3 e^{\phi_0}}{g_{s0}}\left[\frac{k^2}{a^2}+6H^2+\frac{6\ddot{a}}{a}\right]\right\} \\
& +6\dot{\Phi}\left\{12\alpha\sigma_0^2 a^2\left[5H^3e^{-2\phi_0}+\frac{4H\ddot{a}e^{2\phi_0}}{a}\right]+5H\frac{\alpha T_3 e^{\phi_0}}{g_{s0}}\right\}+6\ddot{\Phi}\left\{\frac{\alpha T_3 e^{\phi_0}}{g_{s0}}+12\alpha\sigma_0^2\dot{a}^2e^{-2\phi_0}\right\}\;. \qquad (128)
\end{aligned}
$$

We remind the reader that in our approach we assumed that any dilaton couplings to matter have been suppressed, and this is the reason why no matter contributions appear in the perturbations associated with the dilaton constraint Eq. (128). Here we can see that the dilaton equation specifies the evolution of the metric perturbations entirely. With the form of Φ then inserted in to Eq. (126), the form of ζ is found, and finally with both ζ and Φ used in Eq. (125), $\delta\rho/\rho$ can be calculated.

The numerical results for the matter dominated era, that is a scale factor $a \sim t^{2/3}$, are shown in Figs. 2 and 3, which also shows the effect of altering the D-particle recoil velocity variance, (*cf.* Eq. (56)). A baryon only scenario was considered for the numerical results as an extreme case, in order to isolate the recoil-velocity effects of D-matter. From the figures it can be seen that the magnitude of the variance of the recoil velocity of the D-particles plays the crucial role of allowing matter density perturbations to grow sufficiently to allow for structure formation. In particular, Fig. 3 shows that when the parameter β is sufficiently small, $\beta \leq \mathcal{O}(10^{-4})$, then $\delta\rho/\rho$ begins to show oscillations and no longer exhibits growth. This is to be expected as in the standard gravitational paradigm these same oscillations appear in the absence of dark matter. The role of dark matter is in our case being played by the coupling of the vector field perturbations to $\delta\rho/\rho$. Thus for higher values of β the vector perturbations are strong enough that they can drive structure formation in much the same way that dark matter does in the standard case. Our point is not, however, to do away with dark matter especially as dark matter candidates fall out naturally from

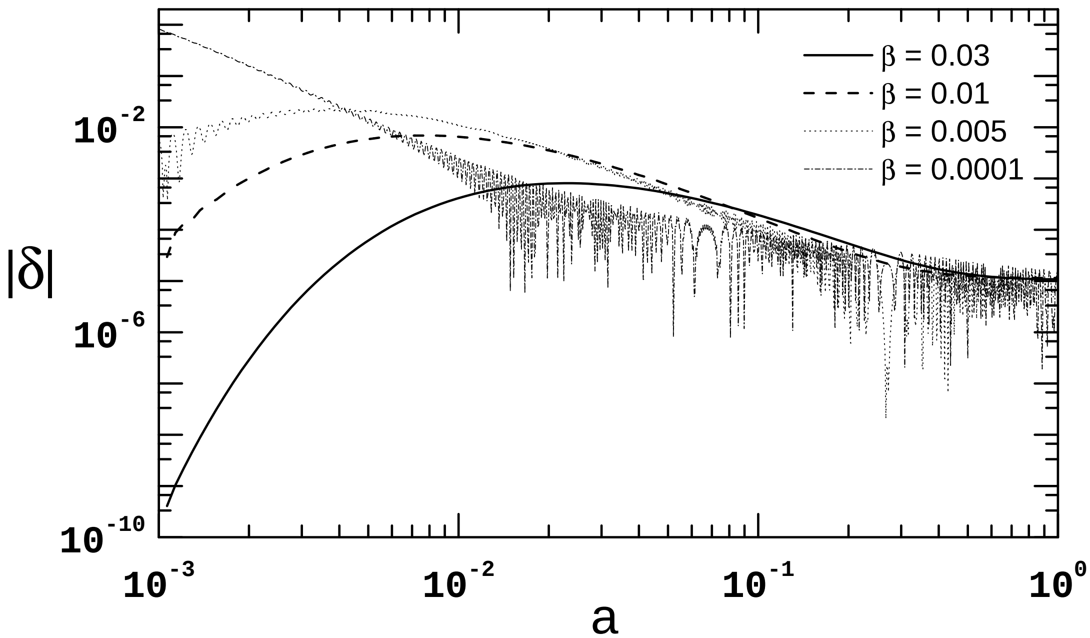

FIG. 3: Over-density parameter $|\delta| = \delta\rho/\rho$ as a function of the scale factor $a(t)$ in the matter-dominated era, for different values of the D-particle recoil velocity variance $\sigma_0^2 \propto \beta/a^3$. The string mass M_s is assumed to be 10 TeV, $\phi_0 = 1$ and $k = 0.5$ Mpc^{-1}. $|\delta|$ is taken to be 10^{-5} today.

the string theory framework our approach relies upon. The point is rather that there may be alternative methods of driving structure formation which are in operation in conjunction with dark matter; the vector perturbations of the approach presented here are one such candidate which has now been shown to have the required characteristics to drive structure formation.

We can also note that the theory is largely insensitive to the string mass, with the critical value of β to induce the growing mode remaining approximately of the same order over the range 10 TeV $< M_s <$ 10^{18} GeV. This can be readily seen by looking once more at the expression for the coefficient b_2 appearing in the perturbed vector equation, Eq. (126). As we noted earlier, only when ϕ_0 is large and negative will those terms proportional to $\alpha = 1/M_s^2$ play a significant role. In the above numerical analysis we took $\phi_0 = 1$, thus these terms have negligible effect. In the background form of the Lagrange multiplier $\langle\lambda(x)\rangle$, given in Eq. (93), we see that the string mass dependence there is actually cancelled by the string mass term appearing in the definition of σ_0^2 given in Eq. (56). Thus $\langle\lambda(x)\rangle$ is independent of the string mass and so the value of β for which the vector field will enter its growing mode is also independent of the string mass.

A further point of interest is the behaviour of the perturbed vector equation in the radiation dominated era. In the radiation dominated era the scale factor $a \sim t^{1/2}$ and as result,

$$\begin{aligned} H^2 + \frac{\ddot{a}}{a} &= 0 \\ 2H^3 - \frac{\ddot{a}H}{a} - \frac{\dddot{a}}{a} &= 0 \end{aligned} \tag{129}$$

This can be used in the expression for the coefficient b_2 in Eq.(127). b_2 now becomes,

$$b_2 = -2\langle\lambda(x)\rangle \ . \tag{130}$$

Thus in the radiation dominated era b_2 is always negative, since $\langle\lambda(x)\rangle > 0$ (*cf.* Eq. (93)), which as mentioned before is the condition for the growth of the vector perturbations. Thus in this era the vector perturbations are always in a growing mode, irrespective of the specific density of the defects, provided of course the latter is non zero and sufficiently large so that the above formalism based on the existence of a recoil-vector field is valid. Once again then the vector perturbations show a behaviour similar to that of dark matter by being able to grow during the radiation dominated era when baryon density perturbation would not be able to cluster due to the radiation pressure. This allows the seeding of the density perturbations which would drive structure formation to begin early in the history of the universe. As the brane Universe enters the matter-dominated era, the growing mode persists only above a critical density of defects, as we have discussed previously (*cf.* Figs. 2,3), which is an interesting feature of the model. Of course in the context of our string models, in this era, other (conventional from a particle physics viewpoint) candidates for dark matter, such as supersymmetric partners of mater excitations or gravitinos do exist and contribute to the dark matter spectra. However, in view of the assumption of sufficiently dense populations of the defects on the brane, required for the "*medium*" interpretation, the cosmological constraints of such models, especially as far as collider searches for supersymmetry are concerned [27], are expected to be modified, depending on the mass range of the D-particles [7].

VII. CONCLUSIONS AND OUTLOOK

In this work, we have analysed some microscopic string theory models of modified gravity arising in the low-energy limit of brane worlds containing space-time point-like brane defects (D-particles). Dense media of such D-particles can exist at early eras of the Universe, without the danger of over-closing the Universe, for specifically stringy reasons. Propagation of neutral matter, such as neutrinos, on such backgrounds leads to effective gravitational theories containing, in addition to the traditional graviton and dilaton fields of the gravitational multiplet of the string, also *vector* gauge fields, describing the recoil of the brane defects during their topologically non-trivial scattering with the string matter. The vector fields are associated with the recoil velocities of the defects and as such satisfy a given constraint Eq. (52). The presence of a recoil velocity locally breaks Lorentz invariance, which however is assumed to be restored on average over large populations of D-particles, since the velocity expectation value vanishes, leaving only the variances of the velocities to be non zero.

The effective Lagrangian describing the low-energy dynamics of this model contains a Dirac-Born-Infeld type of lagrangian for the vector field, coupled non-trivially to space-time curvature. By solving the associated equations of motion we have determined our background configuration, over which perturbations were considered. We have also demonstrated the consistency of the solutions with the conformal invariance conditions of the associated stringy σ-model. The equation of state of the Dirac-Born-Infeld fluid of the vector field has also been considered, and this result can be derived exactly to all orders in α'. The gravitational sector of the model, unlike the vector field, cannot be studied exactly but only perturbatively in powers of α'. In this work we restricted ourselves to considering space-time curvature terms in the effective action up to order $\mathcal{O}(\alpha')$, which suffices for our low-energy considerations at late epochs of the Universe.

By considering perturbations of the vector field, we have shown the possibility of a growing mode in the matter era, for sufficiently large values of the variance of the recoil velocities of the D-particle. The mode also exists in the radiation era. The constraint Eq. (52) of the vector fields is essential in ensuring that the growing mode exists. This allows the seeding of the density perturbations which would drive large-structure formation to begin early in the history of the universe. This feature is shared by TeVeS models in alternative to Dark matter scenarios, but the main difference of our model lies on the fact that in our case the growing mode is independent of the difference of the two gravitational potentials $\Psi - \Phi$ which vanishes here. This is consistent with the equations of motion for the graviton and dilaton fields, and thus the associated conformal invariance conditions of the stringy σ-model. In addition, although in our theory, D-matter plays a role analogous to dark matter as far as large-structure formation is concerned, this is only one component, given that the underlying superstring inspired models do involve additional components coming from the supersymmetric partners of the standard model particle sector of the theory. In this sense, the phenomenology of these models is different from conventional low-energy string-inspired effective theories and can depend crucially on the density of D-particle defects (and the magnitude of their masses $M_{\rm s}/g_{\rm s}$) in the current era. In this spirit, comparison of our class of models against gravitational lensing data is essential in determining the amount of dark matter present in the centre of galaxies, where the concentration of D-matter is expected to be significant.

In our solutions in this article we assumed that the dilaton field was constant. Extensions of our model to incorporate time-dependent dilaton fields are envisaged, although in such a case the reconciliation of the model with the particle physics phenomenology may be subtle, given that the exponential of the dilaton is connected to the string coupling, and the latter to gauge couplings of the low energy field theories coming from the string. If we want standard model

physics to be reproduced today and the findings of the theory of Big Bang Nucleosynthesis not to be disturbed, then one should apply stringent constraints on the varying dilaton fields. Moreover, non constant dilaton fields may have significant impact on the growth of galaxies, and thus severe constraints from the relevant astrophysical data are likely to be imposed, which can easily rule out most of such cosmologies [28].

Finally, we would like to close by stressing the fact that the detailed cosmology of such models is still in its infancy, but we believe that the (rather toy) model we studied here exhibited several interesting and non-trivial features that are worthy of further detailed investigations in more realistic models and in very early eras of the Universe. An interesting aspect we would like to study is the role of the D-particles and the associated vector recoil field in inflation. Such issues may be tackled by considering colliding branes in our context (*cf.* Fig. 1). Brane collision would play the role of a cosmically catastrophic event. During the collision, the concentration of massive bulk D-particles that are trapped in the bulk space between the colliding brane worlds can increase in such a way so that they can collapse to form black holes. The latter would then evaporate, with the standard model particles being trapped on the brane. The gravitational excitations of the string, on the other hand, are capable of escaping in the bulk regions. The collision process can lead to inflationary expansion on the brane universe [29], while the black hole evaporation can contribute to reheating of the inflationary universe and subsequent graceful exit from the inflationary phase. The particular role of the recoil vector field during the interaction of string matter with the defects can be investigated at such very early epochs of the Universe. Although at present these considerations appear to be mere speculations, nevertheless we believe that these issues can be tackled at some detail within the framework of our string-modified gravity. We hope to be able to report progress on some of these topics in a future publication.

A final comment we wish to make concerns the density of D-particles on the brane world. Although cosmologically the later is left unconstrained, as we have already mentioned, given that the issue of overclosure of the brane Universe does not arise, nevertheless, the presence of a finite density medium of D-particles on the brane, with which neutral particles such as neutrinos and photons interact non trivially, implies non-trivial "optical" properties for the brane, such as a *refractive* index. The latter will manifest itself as energy-dependent delays in the arrival times of photons emitted simultaneously from a high-energy astrophysical source, such as a Gamma Ray Burst or Active Galactic Nucleus. As discussed in [14, 30], during each interaction of a high energy neutral particle (represented by a string) with a D-particle, there is a delay in the re-emission process of the particle due to capture which is linear in the incident particle energy E, $\delta t \sim E/M_s^2$, where M_s is the string scale (we work in units of $c = 1$). The total delay turns out to be proportional to the number of D-particle defects per string length, η_{ℓ_s}, encountered by the propagating photon, times the distance travelled, $\Delta t_{\rm total} \sim \frac{\eta_{\ell_s}}{M_s} EL$. This implies an effective suppression scale $M^{\rm eff} = M_s/\eta_{\ell_s}$. The sensitivity of current observations of photons from sources at redshifts $z < 3$ [14, 30] to such linearly suppressed delays is such that $M^{\rm eff}$ is of order of the Planck scale (10^{19} GeV). Since from particle physics experiments we know that $M_s \geq \mathcal{O}(10)$ TeV, we observe that an η_{ℓ_s} as low as 10^{-15} can lead to observable effects from refractive index measurements for low string scales. In our models discussed here, we may easily imagine a depletion of defects in the bulk during epochs $z < 5$, so that their densities on the branes fall below such values, thereby avoiding any constraint from the above refractive index measurements. If our defects on the brane (D-matter) are one or two orders of magnitude more dense that the standard (conventional) dark matter, which is a quite natural situation, these optical constraints will still be negligible. Finally, we remark that the presence of D-matter may affect the peaks of the Cosmic Microwave Background Radiation, depending on the respective densities at that epoch, but this analysis requires a separate study, which goes beyond the purpose of our present article.

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[1] J. D. Bekenstein, Phys. Rev. **D70**, 083509 (2004). [astro-ph/0403694]. For a recent review see: J. D. Bekenstein, In Bertone, G. (ed.): *Particle dark matter*, 99-117. [arXiv:1001.3876 [astro-ph.CO]].

[2] J. W. Moffat, JCAP **0603**, 004 (2006) [gr-qc/0506021]; J. W. Moffat and V. T. Toth, Class. Quant. Grav. **26**, 085002 (2009) [arXiv:0712.1796 [gr-qc]].

[3] M. Milgrom, Astrophys. J. **270**, 365-370 (1983).

[4] D. Clowe, A. Gonzalez, M. Markevitch, Astrophys. J. **604**, 596-603 (2004). [astro-ph/0312273]; I. Ferreras, M. Sakellariadou, M. F. Yusaf, Phys. Rev. Lett. **100**, 031302 (2008). [arXiv:0709.3189 [astro-ph]]; N. E. Mavromatos, M. Sakellariadou, M. F. Yusaf, Phys. Rev. **D79**, 081301 (2009). [arXiv:0901.3932 [astro-ph.GA]]; I. Ferreras, N. E. Mavromatos, M. Sakellari-

adou, M. F. Yusaf, Phys. Rev. **D80**, 103506 (2009). [arXiv:0907.1463 [astro-ph.GA]]; Phys. Rev. **D** in press, arXiv:1205.4880 [astro-ph.CO].
[5] N. Mavromatos and M. Sakellariadou, Phys. Lett. B **652** (2007) 97 [arXiv:hep-th/0703156].
[6] J. R. Ellis, N. E. Mavromatos and M. Westmuckett, Phys. Rev. D **70** (2004) 044036 [arXiv:gr-qc/0405066]; J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and M. Westmuckett, Int. J. Mod. Phys. A **21** (2006) 1379 [arXiv:gr-qc/0508105];
[7] N. E. Mavromatos, V. A. Mitsou, S. Sarkar and A. Vergou, Eur. Phys. J. C **72**, 1956 (2012) [arXiv:1012.4094 [hep-ph]].
[8] J. Polchinski, "*String theory. Vols 1 & 2* " *Cambridge, UK: Univ. Pr. (1998) 402 p.*
[9] C. V. Johnson, "*D-Branes*," *Cambridge, USA: Univ. Pr. (2003) 548 p.*
[10] T. Li, N. E. Mavromatos, D. V. Nanopoulos, D. Xie, Phys. Lett. **B679**, 407-413 (2009). [arXiv:0903.1303 [hep-th]].
[11] S. Dodelson and M. Liguori, Phys. Rev. Lett. **97**, 231301 (2006) [arXiv:astro-ph/0608602].
[12] I. I. Kogan, N. E. Mavromatos, J. F. Wheater, Phys. Lett. **B387**, 483-491 (1996). [hep-th/9606102]; N. E. Mavromatos, R. J. Szabo, Phys. Rev. **D59**, 104018 (1999). [hep-th/9808124]; JHEP **0110**, 027 (2001). [hep-th/0106259].
[13] F. David, Mod. Phys. Lett. **A3**, 1651 (1988); J. Distler, H. Kawai, Nucl. Phys. **B321**, 509 (1989).
[14] N. E. Mavromatos, Int. J. Mod. Phys. **A25**, 5409-5485 (2010). [arXiv:1010.5354 [hep-th]].
[15] See, for instance: A. A. Tseytlin, arXiv:hep-th/9908105, in *Yuri Golfand memorial volume*, ed. M. Shifman, World Scientific, 2000; also in *The many faces of the superworld* (Shifman, M.A. (ed.): 417-452), and references therein.
[16] Y. K. Cheung, M. Laidlaw and K. Savvidy, JHEP **0412**, 028 (2004) [arXiv:hep-th/0406245].
[17] O. D. Andreev, A. A. Tseytlin, Nucl. Phys. **B311**, 205 (1988).
[18] N. Wyllard, Nucl. Phys. B **598**, 247 (2001) [arXiv:hep-th/0008125]; JHEP **0108**, 027 (2001). [hep-th/0107185].
[19] I. Antoniadis, C. Bachas, J. R. Ellis, D. V. Nanopoulos, Nucl. Phys. **B328**, 117-139 (1989).
[20] J. Alexandre, A. Kostouki and N. E. Mavromatos, JHEP **0903**, 022 (2009) [arXiv:0811.4607 [hep-th]].
[21] E. Gravanis, N. E. Mavromatos, JHEP **0206**, 019 (2002).
[22] G. Shiu and L. -T. Wang, Phys. Rev. D **69**, 126007 (2004) [hep-ph/0311228].
[23] E. Elizalde, J. E. Lidsey, S. Nojiri and S. D. Odintsov, Phys. Lett. B **574**, 1 (2003) [hep-th/0307177].
[24] N. E. Mavromatos, Phys. Rev. D **83**, 025018 (2011) [arXiv:1011.3528 [hep-ph]].
[25] C. Skordis, Phys. Rev. **D74**, 103513 (2006). [astro-ph/0511591].
[26] M. Feix, arXiv:1203.2891 [gr-qc].
[27] For reviews see, e.g.: A. B. Lahanas, N. E. Mavromatos and D. V. Nanopoulos, Int. J. Mod. Phys. D **12**, 1529 (2003) [hep-ph/0308251]; C. Munoz, Int. J. Mod. Phys. A **19**, 3093 (2004) [hep-ph/0309346], and references therein.
[28] S. Basilakos, N. E. Mavromatos, V. A. Mitsou and M. Plionis, Astropart. Phys. **36**, 7 (2012) [arXiv:1107.3532 [astro-ph.CO]].
[29] J. R. Ellis, N. E. Mavromatos, D. V. Nanopoulos and A. Sakharov, New J. Phys. **6**, 171 (2004) [gr-qc/0407089].
[30] J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Lett. B **665**, 412 (2008) [arXiv:0804.3566 [hep-th]]; Phys. Lett. B **694**, 61 (2010) [arXiv:1004.4167 [astro-ph.HE]].